Axiomatic characterizations and a non-cooperative justification of the constrained equal benefits rule in airport problems

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Abstract

We propose a converse consistency property and study its implications, when imposed together with other properties, in airport problems. We find that the constrained equal benefits rule satisfies our converse consistency, and then offer axiomatic arguments in favor of the rule on the basis of the property. In addition, we use consistency and our converse consistency as guides to design an extensive form game, and show that the unique subgame perfect equilibrium outcome of the game is the contribution vector suggested by the CEB rule, which gives a non-cooperative justification of the rule. *Journal of Economic Literature* Classification Numbers: C72; D63.

Keywords: Bilateral consistency; converse consistency; the constrained equal benefits rule; modified nucleolus; airport problems

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1 Introduction

We consider the problem of sharing the cost of a landing strip among airlines who need airstrips of different lengths. A "rule" is a function that associates with each airport problem, an allocation of the cost of the airstrip, which we call a "contribution vector." A number of desirable properties of rules have been formulated for this problem, and motivated by various fairness principles. The literature devoted to the search for rules satisfying these properties, singly and in various combinations, is initiated by Littlechild and Owen (1973).[1]

Our first purpose is to define the bilateral version of the following consistency property[2], and then propose the converse of the bilateral consistency property and study its implications when imposed together with other desirable properties. Let c_i be the cost parameter that represents the cost of fulfilling airline i's need. Consider a problem and a contribution vector x chosen by a rule for it. Imagine that some airlines pay their contributions and "leaves", and reassess the situation from the viewpoint of the remaining airlines. Let the cost parameters of the remaining airlines be revised as follows: (i) for each airline j, its revised cost parameter is the maximum of 0 and $c_j - x_i$. Then, "consistency" (Potters and Sudhölter, 1999) of the rule says that the components of x pertaining to the remaining airlines should still be chosen by the rule for the reduced problem just defined.[3]

We say that a rule is "conversely consistent" if whenever for some airport problem, a contribution vector has the property that for each proper two-airline subgroups, the rule chooses the restriction of the vector to this subgroup for the reduced problem it faces, then the vector should be recommended for the initial problem by the rule.[4] We show that the so-called

[1]For a comprehensive survey of this literature, see Thomson (2004).

[2]This property is an application of a general principle of consistency for airport problem. The general consistency principle has been studied extensively in a great variety of models such as taxation, bargaining, exchange, matching, social choice *etc.* For a comprehensive survey of the literature on this principle, see Thomson (2000).

[3]Potters and Sudhölter (1999) refer to it as ψ*-consistency*. We adopt Thomson (2004)'s terminology.

[4]Our *converse consistency* is an application of a general principle of converse consistency for airport problems. The general principle of converse consistency has been studied and applied to a number of models by many authors such as Chang and Hu (2007); Chun (2002).

"constrained equal benefits rule"[5] (henceafter, the CEB rule), which equates airlines' benefits (the difference between the cost of satisfying this airline's need and his contribution) subject to no one receiving a transfer, satisfies *converse consistency*. We then base axiomatic characterizations of the CEB rule on this property together with other desirable properties.

Our second purpose is to justify the CEB rule from the non-cooperative perspective. It is well-known that the non-cooperative justification for a rule often follows from designing a specific bargaining procedure that leads to the outcome of the rule.[6] Like several others[7], we use *consistency* and *converse consistency* as guides in designing an extensive form game. We show that the unique subgame perfect equilibrium outcome of the game is the contribution vector recommended by the CEB rule, which gives a non-cooperative justification of the rule.

2 Notation and definitions

2.1 The model

Let $U \subsetneqq \mathbb{N}$ be a universe of agents with at least two elements, where $\mathbb{N}$ is the set of natural numbers. Given a finite and nonempty set $N \subsetneqq U$ and $i \in N$, let $c_i \in \mathbb{R}_+$ be **agent *i*'s cost parameter** and $c = (c_i)_{i \in N} \in \mathbb{R}^N_+$ be the profile of cost parameters. An airport problem, or simply a **problem**, is a pair (N, c). Let $\mathcal{A}$ be the class of all problems on U. Given $(N, c) \in \mathcal{A}$, let n denote the number of agents in N. Without loss of generality, we assume that $N \equiv \{1, \dots, n\}$ and $c_1 \leq \cdots \leq c_n$. Thus, we refer to agent 1 as the first agent and agent n as the last agent. A **contribution vector** for $(N, c) \in \mathcal{A}$ is a vector $x \in \mathbb{R}^N_+$ such that $\sum_{i \in N} x_i = \max_{i \in N} c_i$ and for each $i \in N$, $x_i \leq c_i$. Let $X(N, c)$ be the set of all contribution vectors for $(N, c) \in \mathcal{A}$. A **rule** is a function defined on $\mathcal{A}$ that associates with each problem $(N, c) \in \mathcal{A}$ a vector $x \in X(N, c)$. Our generic notation for rules is φ. For each coalition $N' \subset N$, we denote $(c_i)_{i \in N'}$ by $c_{N'}$, $(\varphi_i(N, c))_{i \in N'}$ by $\varphi_{N'}(N, c)$, and so on.

[5] The rule is introduced by Sudhölter (1996, 1997) under the name of modified nucleolus.

[6] For this line of research, see Serrano (1997), Krishna and Serrano (1996), and Serrano (2005).

[7] For references, see Dagan et al. (1997); Chang and Hu (2008).

2.2 The central rules and properties

We now introduce our central rules. The first one is defined only for the two-agent problem. It says that the agent with smaller cost parameter contributes half of his cost parameter, and the other contributes the remaining amount to be collected.

Standard rule, $\boldsymbol{S}$: For each $i, j \in N$ and each $(\{i,j\},(c_i,c_j)) \in \mathcal{A}$ with $c_i \leq c_j$,

$$\begin{aligned} S_i(\{i,j\},(c_i,c_j)) &= \tfrac{c_i}{2} \\ S_j(\{i,j\},(c_i,c_j)) &= c_j - S_i(\{i,j\},(c_i,c_j)). \end{aligned}$$

The following rule, which is proposed by Sudhölter (1996, 1997) under the name of modified nucleolus in the theory of transferable utility game, was informally introduced in Section 1. The terminology we adopt here is borrowed from Thomson (2004).

Constrained Equal Benefits rule, $\boldsymbol{CEB}$: For each $(N,c) \in \mathcal{A}$ and each $i \in N$,

$$CEB_i(N,c) \equiv \max\{c_i - \beta, 0\},$$

where $\beta \in \mathbb{R}_+$ is chosen such that $\sum_{i\in N} CEB_i(N,c) = \max_{j\in N} c_j$.

Remark 1: It is known that the CEB rule, the "sequential equal contributions rule", and the "nucleolus" coincide with the standard rule for two-agent problems.

We next formulate our central properties. The first one is the bilateral version of the following consistency property. Consider a problem and a contribution vector x chosen by a rule for this problem. Imagine now that agent i contributes x_i and leaves, and reassess the situation from the viewpoint of the remaining agents. Instead of thinking of x_i as covering an abstract part of the airstrip, it is natural to think of x_i as a contribution to the part of the airstrip agent i uses. For each agent j (with $c_j \geq c_i$), its cost parameter is revised down by the amount x_i since contributing to the part of the airstrip agent i uses implies contributing to the part of the airstrip that agents i and j use. For each agent k (with $c_k < c_i$), how should x_i be imputed to agent k's cost parameter? One can think of x_i as a contribution to the part of the airstrip that agents i and k use. If $c_k > x_i$, then agent k benefits completely from agent i's contribution. Thus, agent k's cost parameter is revised down by the amount x_i; otherwise, agent k benefits partially from agent i's contribution. Since c_k is the maximal amount that agent k can benefit, agent k's cost parameter is revised down by the amount c_k. Namely, the revised cost parameter of agent k is zero. Thus, the revised cost parameter of agent k is obtained by taking the maximum of zero and $c_k - x_i$. "Consistency" (Potters and Sudhölter, 1999) of the rule says that the components of x pertaining to the remaining agents should still be chosen by the rule for the reduced problem just defined.[8]

We apply the above idea to define the bilateral version of *consistency*. Let $(N, c) \in \mathcal{A}$ with $n \geq 2$, $i \in N \backslash \{n\}$, and $x \in X(N, c)$. The **reduced problem of $\boldsymbol{(N, c)}$ with respect to $\boldsymbol{N' = \{i, n\}}$ and $\boldsymbol{x}$**, $(N', uc^x_{N'})$, is defined by[9]

$$(uc^x_{N'})_i = \max \left\{ c_i - \textstyle\sum_{k \neq i,n} x_k, 0 \right\}, \text{ and}$$

$$(uc^x_{N'})_n = \max\{c_n - \textstyle\sum_{k \neq i,n} x_k, 0\}.$$

Bilateral consistency: For each $(N, c) \in \mathcal{A}$ with $n \geq 2$ and each $i \in N \backslash \{n\}$, if $x = \varphi(N, c)$, then $\left(\{i, n\}, uc^x_{\{i,n\}}\right) \in \mathcal{A}$ and $x_{\{i,n\}} = \varphi\left(\{i, n\}, uc^x_{\{i,n\}}\right)$.

We next formulate the converse version of *bilateral consistency*.[10]

Converse consistency: For each $(N, c) \in \mathcal{A}$ with $n > 2$ and each $x \in X(N, c)$, if for each $N' \subset N$ with $|N'| = 2$ and $n \in N'$, $x_{N'} = \varphi(N', uc^x_{N'})$, then $x = \varphi(N, c)$.

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[8] Potters and Sudhölter (1999) do not consider the possibility of the departure of the last agent in the formulation of *consistency* since c_n determines the total cost and after the departure of the last agent, the new total cost to be covered would have no reason to be related to the sum of the contributions required of the remaining agents.

[9] Potters and Sudhölter (1999) define their reduced problem as follows: Let $(N, c) \in \mathcal{A}$ with $n \geq 2$, $i \in N \backslash \{n\}$, and $x \in X(N, c)$. The reduced problem of (N, c) with respect to $N' = N \backslash \{i\}$ and x, $(N', uc^x_{N'})$, is defined as follows: for each $j \in N'$ such that $c_j < c_i$, $(uc^x_{N'})_j = \max\{c_j - x_i, 0\}$, and for each $j \in N'$ such that $c_j \geq c_i$, $(uc^x_{N'})_j = c_j - x_i$.

[10] It can be shown that *bilateral consistency* and *converse consistency* are logically independent.

3 Axiomatic characterizations

The following lemma establishes the existences of our axiomatic characterizations of the CEB rule.

Lemma 1: The CEB rules satisfies *converse consistency*.

Proof. Let $(N,c) \in \mathcal{A}$. Assume that $N = \{1, \cdots, n\}$ and $c_1 \leq \cdots \leq c_n$. Let $x \in X(N,c)$ be such that for each $i \neq n$, $x_{\{i,n\}} = CEB\left(\{i,n\}, uc^x_{\{i,n\}}\right)$. We show that $x = CEB(N,c)$. Let $i \in N \setminus \{n\}$. Note that $\sum\limits_{k \in N} x_k = c_n$ and $x_{\{i,n\}} = CEB\left(\{i,n\}, uc^x_{\{i,n\}}\right)$. It follows that $\left(uc^x_{\{i,n\}}\right)_n = x_i + x_n$ and $\left(uc^x_{\{i,n\}}\right)_i = 2x_i$. Let $\beta = c_n - x_n$. Since $c_n = \sum_{k \in N} x_k$, then

$$c_i - \beta = c_i - c_n + x_n = \left(c_i - \sum_{k \neq i,n} x_k\right) - x_i.$$

If $c_i - \sum\limits_{k \neq i,n} x_k \geq 0$, then $2x_i = \left(uc^x_{\{i,n\}}\right)_i = c_i - \sum\limits_{k \neq i,n} x_k$. It follows that $\max\{c_i - \beta, 0\} = x_i$. If $c_i - \sum\limits_{k \neq i,n} x_k < 0$, then $\left(uc^x_{\{i,n\}}\right)_i = 0$. Thus, $x_i = 0$. It follows that $c_i - \beta \leq 0$. Thus, $\max\{c_i - \beta, 0\} = 0 = x_i$. We then conclude that $x = CEB(N,c)$. *Q.E.D.*

We consider the following desirable properties. The first one says that two agents with the same cost parameters should contribute equal amounts.

Equal treatment of equals: For each $N \in \mathcal{N}$, each $c \in \mathcal{C}^N$, and each pair $\{i,j\} \subseteq N$, if $c_i = c_j$, then $\varphi_i(N,c) = \varphi_j(N,c)$.

The second one is an order property. It says that of two agents, the benefit (the difference between the cost parameter of an agent and this agent's contribution) to the agent with the larger cost parameter should be at least as much as that to the other.[11]

Order preservation for benefits: For each $N \in \mathcal{N}$, each $c \in \mathcal{C}^N$, and each pair $\{i,j\} \subseteq N$, if $c_i \leq c_j$, then $c_i - \varphi_i(N,c) \leq c_j - \varphi_j(N,c)$.

[11]The property is introduced by Littlechild and Thompson (1977).

Our next property has to do with a possible increase of the cost parameter of the last agent. It says that if the cost parameter of the last agent increases by δ, then this agent's contribution should increase by δ.[12]

Last-agent cost additivity: For each pair $\{(N,c),(N,c')\}$ of elements of $\mathcal{A}$ and each $\delta \in \mathbb{R}_+$, if $c'_n = c_n + \delta$ and for each $j \in N\backslash\{n\}$, $c'_j = c_j$, then $\varphi_n(N,c') = \varphi_n(N,c) + \delta$.

The last one is a monotonicity requirement. It says that if an agent's cost parameter increases, all other agents should contribute at most as much as they did initially.[13]

Cost monotonicity: For each $N \in \mathcal{N}$, each $c \in \mathcal{C}^N$, each $c' \in \mathcal{C}^N$, and each $i \in N$, if $c'_i \geq c_i$ and for each $j \in N\backslash\{i\}$, $c'_j = c_j$, then for each $j \in N\backslash\{i\}$, $\varphi_j(N,c') \leq \varphi_j(N,c)$.

Remark 2: It is clear that *order preservation for benefits* implies *equal treatment of equals*.

We are now ready to present the two announced characterizations of the standard rule, which play important roles in our paper.

Proposition 1: For $|N| = 2$. The standard rule is the only rule satisfying *order preservation for benefits* and *cost monotonicity*.

Proof. It is clear that the standard rule satisfies the two properties. Conversely, let $(N,c) \in \mathcal{A}$ with $N \equiv \{i,j\}$ and $c_i \leq c_j$. Let φ be a rule satisfying the two properties. Let $x = \varphi(N,c)$ and $y = S(N,c)$. We show that $x_i = y_i$. By *efficiency*, we then conclude that $x = y$. We consider two cases.

Case 1: $c_i = c_j$. By Remark 2, *order preservation* implies *equal treatment of equals*. Thus, $x_i = x_j = \frac{c_i}{2} = y_i$.

Case 2: $c_i < c_j$. Note that for the two-agent case, *order preservation for benefits* implies that $x_i \geq \frac{c_i}{2}$. We show that $x_i = \frac{c_i}{2}$. Let $c' \in \mathcal{A}$ with

[12] It is a weaker version of the property, "covariance", appeared in Potters and Sudhölter (1999).

[13] This property is introduced by Thomson (2004) under the name of "others-oriented cost monotonicity". It is a complement of "individual cost monotonicity" (Potters and Sudhölter, 1999) and says the following. Under the same hypotheses, agent i should pay at least as much as he did initially.

$N \equiv \{i, j\}$ and $c'_i = c_i$ and $c'_j = c_i$. Let $x' = \varphi(N, c')$. By Case 1, $x'_i = \frac{c_i}{2}$. Note that $c'_i = c_i$ and $c'_j < c_j$. By *cost monotonicity*, $x_i \leq x'_i$. Thus, $x_i \leq \frac{c_i}{2}$ and we derive $x_i = \frac{c_i}{2}$. *Q.E.D.*

Proposition 2: For $|N| = 2$. The standard rule is the only rule satisfying *equal treatment of equals* and *last-agent cost additivity*.

Proof. It is clear that the standard rule satisfies the two properties. Conversely, let $(N, c) \in \mathcal{A}$ with $N \equiv \{i, j\}$ and $c_i \leq c_j$. Let φ be a rule satisfying the two properties. Let $x = \varphi(N, c)$ and $y = S(N, c)$. We show that $x = y$. We consider two cases.

Case 1: $c_i = c_j$. By *equal tretament of equals*, $x_i = x_j$. By *efficiency*, $x_i = x_j = \frac{c_i}{2} = y_i = y_j$.

Case 2: $c_i < c_j$. Let $c' \in \mathcal{A}$ with $N \equiv \{i, j\}$ and $c'_i = c_i$ and $c'_j = c_i$. Let $x' = \varphi(N, c')$. By Case 1, $x'_j = \frac{c_i}{2}$. By *last-agent cost additivity* of the CEB rule, $x_j = x'_j + \delta$, where $\delta \equiv c_j - c_i$. Then, $x = y$. *Q.E.D.*

We use the following lemma to derive our characterizations of the CEB rule.

Elevator Lemma (Thomson, 2000): If a rule φ is *consistent* and coincides with a *conversely consistent* rule φ' in the two-agent case, then φ coincides with φ' in general.

It can be shown that the CEB rule satisfies *equal treatment of equals*, *order preservation for benefits*, *cost monotonicity*, *last-agent cost additivity*, and *bilateral consistency*. As shown in Lemma 1, it satisfies *converse consistency*. With the help of the Elevator Lemma and Propositions 1 and 2, the next results are immediate.[14]

Theorem 1: The CEB rule is the only *bilaterally consistent* rule satisfying *order preservation for benefits* and *cost monotonicity*.

[14]Note that in the formulations of *bilateral consistency* and *converse consistency*, we do not consider all two-agent subgroups. Thus, Theorems 1, 2, 3, and 4 are special applications of the Elevator Lemma. To illustrate how the lemma can be applied here, we give a formal proof of Theorem 1. Let $(N, c) \in \mathcal{A}$ with $N \equiv \{1, \ldots, n\}$ and $c_1 \leq \cdots \leq c_n$. Let φ be a *bilaterally consistent* rule satisfying the two properties. Let $x = \varphi(N, c)$ and $y = CEB(N, c)$. We show that $x = y$. By *bilateral consistency* of φ, for each $i \in N \backslash \{n\}$, $\varphi_{\{i,n\}}(N, c) = x_{\{i,n\}}$. Note that by Proposition 1, $x_{\{i,n\}} = y_{\{i,n\}}$. By *converse consistency* of the CEB rule, we conclude that $x = y$. *Q.E.D.*

Theorem 2: The CEB rule is the only *conversely consistent* rule satisfying *order preservation for benefits* and *cost monotonicity*.

Theorem 3: The CEB rule is the only *bilaterally consistent* rule satisfying *equal treatment of equals* and *last-agent cost additivity*.

Theorem 4: The CEB rule is the only *conversely consistent* rule satisfying *equal treatment of equals* and *last-agent cost additivity*.

Hu, Tsay, and Yeh (2010) consider the bilateral version of another consistency concept, "ν-consistency", introduced by Potters and Sudhölter (1999). The authors show that the "nucleolus" is the only *bilaterally ν-consistent* or *conversely ν-consistent* rule satisfying *equal treatment of equals* and *last-agent cost additivity*. Their characterizations of the nucleolus together with our Theorems 3 and 4 illustrate that *consistency* and *converse consistency* play important roles to distinguish the nucleolus and the CEB rule.

Potters and Sudhölter (1999) show that the CEB rule is the only *consistent* rule satisfying *equal treatment of equals* and the following two properties. The first one, "homogeneity", says that if all cost parameters are multiplied by the same positive number, so should the contributions. The second one, "strong last-agent cost additivity", says that if the cost parameter of an agent with the largest cost parameter increases by δ, then its contribution should increase by δ, and all other agents should contribute the same amounts as they did initially.[15] Theorem 3 and Potters and Sudhölter (1999) characterization provide axiomatic justification for the CEB rule. In that sense, the two results are closely related. To compare the two results, note that we impose, "efficiency": the sum of the contributions should be equal to the entire cost, on the definition of a rule. Thus, Theorem 3 says that a characterization of the CEB rule can still be obtained from Potters and Sudhölter (1999) characterization by replacing *homogeneity* by *efficiency*, and weakening *consistency* and *strong last-agent cost additivity* to *bilateral consistency* and *last-agent cost additivity*, respectively. As pointed out by Potters and Sudhölter (1999), *homogeneity*, *strong last-agent cost additivity* and *consistency* altogether imply *efficiency*. In addition, *strong last-agent cost additivity* and *consistency* are stronger than *last-agent cost additivity* and *bilateral consistency*, respectively. Thus, our Theorem 3 implies Potters and Sudhölter (1999) result (check this).

[15] Potters and Sudhölter (1999) combine *strong last-agent cost additivity* and *homogeneity* as a property, referred to as *covariance*.

4 Non-cooperative justification

We next establish a non-cooperative foundation of the CEB rule. It is well-known that a non-cooperative justification of a rule often follows from designing a particular bargaining process or game form that leads to the outcome of the rule. We follow Krishna and Serrano (1996)'s idea to make use of the properties of the solution concept, particularly consistency property or its converse, to design the game form. Dagan et al. (1997), and Chang and Hu (2008) study a specific class of the f-just rules of bankruptcy problems and provide additional supports for the idea.

Let (N, c) be a problem. We design the following 3-stage extensive form game $\Gamma(N, c)$ that captures the notions of both *bilateral consistency* and *converse consistency*. The game tree of $\Gamma(N, c)$ is depicted in Figure 1.

***Stage 1*:** Each agent $k \in N\backslash\{n\}$ proposes his/her own voluntary contribution $x_k \in \mathbb{R}_+$ with $0 \leq x_k \leq c_k$. Let $x_n = c_n - \sum_{k \neq n} x_k$, which we call the induced contribution of agent n. We refer to $x = (x_k)_{k \in N}$ as the *binding proposal*.

***Stage 2*:** Agent n decides to take either action A (accept x_n) or action R (reject x_n). If A is taken, then agent n contributes x_n and the game ends. The binding proposal x is the outcome of the game. If R is taken, then agent n takes an agent in $N\backslash\{n\}$, say agent i, with him to the next stage where their contributions will be determined. The contribution of each agent $k \in N\backslash\{i, n\}$ is equal to his/her voluntary contribution x_k.

***Stage 3*:** *Nature* picks one agent between agents i and n at random with equal probability. If agent i is chosen, then the game ends. Agent i contributes $z_i = \max\left\{0, c_i - \sum_{k \neq i,n} x_k\right\}$ and agent n contributes the residual $z_n = x_i + x_n - z_i$. If agent n is chosen, then the game ends. Agent n contributes $z_n = \max\left\{0, c_n - \sum_{k \neq i,n} x_k\right\}$ and agent i contributes the residual $z_i = x_i + x_n - z_n$.

We next explain how the game $\Gamma(N, c)$ captures the notions of both *bilateral consistency* and *converse consistency*, and offer some interesting observations.

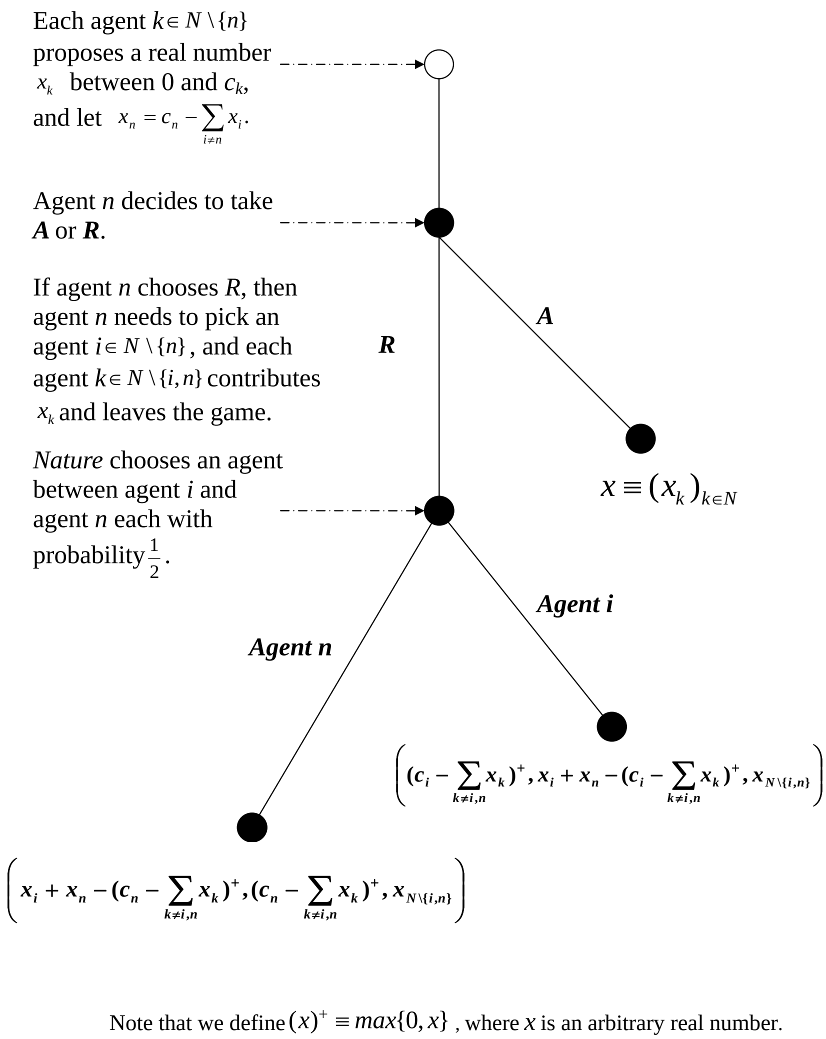

Figure 1: The game tree of $\Gamma(N, c)$

1. In Stage 1, each agent $k \in N \backslash \{n\}$ announces his/her own voluntary contribution. One may wonder why agent n plays no role in this stage. It is because we make use of *bilateral consistency* to design the game $\Gamma(N, c)$. This idea is suggested by Krishna and Serrano (1996) in Nash bargaining problems. Recall that the definition of *bilateral consistency*, agent n is required to be in each possible reduced problem. In addition, the requirement that the voluntary contribution of each agent, except agent n, should be bounded above by his/her cost parameter and below by 0, is reasonable. Note that there is no restriction on the induced contribution of agent n. Namely, x_n could be negative or greater than c_n. However, as we show, the equilibrium contribution of agent n will be indeed bounded above by c_n and below by 0.

2. Dagan et al. (1997), and Chang and Hu (2008) study non-cooperative foundations of bankruptcy rules. In their settings, each agent is required to propose not only his own payoff but also the others' payoffs. However, in our setting, each agent is required to propose his own payoff only. Thus, the information requirement for each agent in our setting is less demanding.

3. In Stage 3, agents i and n play no role. Their contributions will be determined by a randomization process which we call *Nature*. If *Nature* chooses agent i, then the game ends and the contribution of agent i is designed to be $\max\left\{0, c_i - \sum_{k \neq i,n} x_k\right\}$. In other words, agent i compares c_i and $\sum_{k \neq i,n} x_k$. If $c_i > \sum_{k \neq i,n} x_k$, then the amount $\sum_{k \neq i,n} x_k$ is not enough to build an airstrip that agent i can use. To fulfill agent i's need, agent i will contribute the difference $c_i - \sum_{k \neq i,n} x_k$. In this case, agent n contributes $x_i + x_n - \left(c_i - \sum_{k \neq i,n} x_k\right)$. Namely, $c_n - c_i$; otherwise, agent i contributes nothing. Thus, agent n contributes $x_i + x_n$. Similar argument can be applied to the case when *Nature* chooses agent n.

4. Combining (1) and (3), we can see that $\Gamma(N, c)$ captures the notions of both *bilateral consistency* and *converse consistency.*

5. In Stage 3, we introduce *Nature* to determine the contributions of agents i and n, and consider agents' expected equilibrium contributions. In the literature on Nash program, several authors, such as Hart

and Mas-Colell (1996) and Serrano (1997), also adopt similar randomization process to design agents' payoffs and consider agents' expected equilibrium payoffs.

6. Dagan et al. (1997) and Chang and Hu (2008) provide non-cooperative justifications for a class of f-just rules in bankruptcy problems. Their results rely on a certain solution to solve two-agent bankruptcy problems. In contrast, our results do not invoke any solution to solve two-agent airport problems.

The following notation is useful to construct a subgame perfect equilibrium of the game $\Gamma(N,c)$. Let $x \in \mathbb{R}^N$ be a binding proposal. Suppose that agent n picks an agent, say agent $i \in N\backslash\{n\}$, in Stage 2. We denote the expected contributions of agents i and n in Stage 3 by $\tau^i(c,x) = (\tau_i^i(c,x), \tau_n^i(c,x))$, where

$$\tau_i^i(c,x) = \frac{1}{2}\left\{\max\{0, c_i - \sum_{k\neq i,n} x_k\} + x_i + x_n - \max\{0, c_n - \sum_{k\neq i,n} x_k\}\right\}$$

and

$$\tau_n^i(c,x) = x_i + x_n - \tau_i^i(c,x).$$

As we shown next, if $x \in X(N,c)$ and agent n picks agent i in Stage 2, then $\tau^i(c,x) = CEB\left(\{i,n\}, uc^x_{\{i,n\}}\right)$.

Lemma 2: Let $(N,v) \in \mathcal{A}$. If the binding proposal x is a contribution vector in $X(N,c)$, and agent n chooses agent i in Stage 2, then $\tau^i(c,x) = CEB\left(\{i,n\}, uc^x_{\{i,n\}}\right)$.

Proof. Let $\beta = \frac{1}{2}\left(uc^x_{\{i,n\}}\right)_i$ and $x \in X(N,c)$. Thus, $\left(uc^x_{\{i,n\}}\right)_n = \max\{0, c_n - \sum_{k\neq i,n} x_k\} = x_i + x_n$. Note that the sum of the expected contributions of agents i and n is equal to $x_i + x_n$. Thus, it suffices to show that agent i's expected contribution is equal to $CEB_i\left(\{i,n\}, uc^x_{\{i,n\}}\right)$. To see this, note that agent i's expected contribution is

$$
\begin{aligned}
&\frac{1}{2}\left\{\left(uc^{x}_{\{i,n\}}\right)_i + x_i + x_n - \left(uc^{x}_{\{i,n\}}\right)_n\right\}\\
&= \frac{1}{2}\left(uc^{x}_{\{i,n\}}\right)_i\\
&= \left(uc^{x}_{\{i,n\}}\right)_i - \beta\\
&= CEB_i\left(\{i,n\}, uc^{x}_{\{i,n\}}\right).
\end{aligned}
$$

Q.E.D.

The next two results establish our non-cooperative justification of the CEB rule. The first one says that the contribution vector recommended by the CEB rule can be supported by a subgame perfect equilibrium of the game $\Gamma(N,c)$. As usual, we solve the game $\Gamma(N,c)$ by backward induction.

Theorem 5: There exists a subgame perfect equilibrium of $\Gamma(N,c)$ with outcome $CEB(N,c)$.

Proof. The proof is by construction of a strategy profile that is a subgame perfect equilibrium of the game $\Gamma(N,c)$ and generates outcome $CEB(N,c)$. Let f be the strategy profile defined as follows:

- Stage 1: Each agent $k \in N\backslash\{n\}$ proposes $CEB_k(N,c)$.
- Stage 2: Assume that x is a binding proposal. Let $\mu = \min_{k\in N\backslash\{n\}} \tau^k_n(c,x)$. Agent n decides to take action A or action R. If $x_n \leq \mu$, then agent n takes action A. If $x_n > \mu$, then agent n takes action R and picks agent $i \in N\backslash\{n\}$ with $\tau^i_n(c,x) = \mu$.

Step 1: The outcome of the strategy profile $\boldsymbol{f}$ of the game $\boldsymbol{\Gamma(N,c)}$ is $\boldsymbol{CEB(N,c)}$. Each agent $k \neq n$, by following f_k, proposes $CEB_k(N,c)$ in Stage 1. Let $\bar{x} = CEB(N,c)$. Then $\bar{x}$ is a binding proposal. By *bilateral consistency* of the CEB rule, for each $k \neq n$,

$$
\begin{aligned}
\bar{x}_n &= CEB_n\left(\{k,n\}, uc^{\bar{x}}_{\{k,n\}}\right)\\
&= \tau^k_n(c,\bar{x}).
\end{aligned}
$$

Thus, $\mu = \bar{x}_n$. By following f_n, agent n then takes action A. The game ends with outcome $CEB(N,c)$.

Step 2: The strategy profile $\boldsymbol{f}$ is a subgame perfect equilibrium of the game $\boldsymbol{\Gamma(N,c)}$. It is easy to verify that following the strategy f_n is the best response for agent n in Stage 2 provided each agent $k \neq n$ follows f_k. Let $i \in N\backslash\{n\}$ and suppose that each agent $k \in N\backslash\{i\}$ follows f_k. Suppose that in Stage 1, agent i deviates by proposing $x_i' = \bar{x}_i - \varepsilon$ for some $\varepsilon > 0$. Let x' be the binding proposal induced by agent i's deviation. Note that $\bar{x}_i + \bar{x}_n \geq 0$, $x_n' = \bar{x}_n + \varepsilon$, and for each $k \in N\backslash\{i,n\}$, $x_k' = \bar{x}_k$. Note that $\bar{x} \in X(N,c)$. By Lemma 2, $\tau_i^i(c,\bar{x}) = \bar{x}_i$. Thus,

$$
\begin{aligned}
\tau_i^i(c,x') &= \frac{1}{2}\{\max\{0, c_i - \sum_{l\neq i,n} x_l'\} + x_i' + x_n' - \max\{0, c_n - \sum_{l\neq i,n} x_l'\}\} \\
&= \frac{1}{2}\{\max\{0, c_i - \sum_{l\neq i,n} \bar{x}_l\} + \bar{x}_i + \bar{x}_n - \max\{0, c_n - \sum_{l\neq i,n} \bar{x}_l\}\} \\
&= \tau_i^i(c,\bar{x}) \\
&= \bar{x}_i.
\end{aligned}
$$

Note that $\tau_n^i(c,x') = x_i' + x_n' - \tau_i^i(c,x')$. It follows that $\tau_n^i(c,x') = \bar{x}_n$. Let $k \in N\backslash\{i,n\}$. Note that $x_k' + x_n' \geq 0$. Thus,

$$
\begin{aligned}
\tau_k^k(c,x') &= \frac{1}{2}\{\max\{0, c_k - \sum_{l\neq k,n} x_l'\} + x_k' + x_n' - \max\{0, c_n - \sum_{l\neq k,n} x_l'\}\} \\
&= \frac{1}{2}\max\{0, c_k - \sum_{l\neq k,n} \bar{x}_l + \varepsilon\}.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\tau_n^k(c,x') &= x_k' + x_n' - \tau_k^k(c,x') \\
&= \bar{x}_k + \bar{x}_n + \varepsilon - \frac{1}{2}\max\{0, c_k - \sum_{l\neq k,n} \bar{x}_l + \varepsilon\}.
\end{aligned}
$$

If $c_k - \sum_{l\neq k,n} \bar{x}_l + \varepsilon \leq 0$, then $\tau_n^k(c,x') \geq \bar{x}_n + \frac{1}{2}\varepsilon > \bar{x}_n = \tau_n^i(c,x')$. If $c_k - \sum_{l\neq k,n} \bar{x}_l + \varepsilon > 0$, then by *order preservation for benefits* of the CEB rule,

$$
\begin{aligned}
\tau_n^k(c,x') &= \bar{x}_k + \bar{x}_n + \varepsilon - \frac{1}{2}\max\{0, c_k - \sum_{l\neq k,n} \bar{x}_l + \varepsilon\} \\
&= \bar{x}_n + \frac{\varepsilon}{2} + \frac{1}{2}\left\{\bar{x}_k + \sum_{l\neq n} \bar{x}_l - c_k\right\} \\
&= \bar{x}_n + \frac{\varepsilon}{2} + \frac{1}{2}\{c_n - \bar{x}_n - (c_k - \bar{x}_k)\} \\
&\geq \bar{x}_n + \frac{\varepsilon}{2} \\
&> \bar{x}_n \\
&= \tau_n^i(c,x').
\end{aligned}
$$

By following f_n in Stage 2, agent n then picks agent i. Then, agent i ends up with the contribution $\bar{x}_i$. Thus, agent i is not better off by deviating. Suppose that in Stage 1, agent i deviates by proposing $x_i' = \bar{x}_i + \varepsilon$ for some $\varepsilon > 0$. If in Stage 2, agent n takes action A, or if in Stage 2, agent n takes action R and picks agent $k \in N\setminus\{i,n\}$, then agent i ends up with the contribution $\bar{x}_i + \varepsilon$. Thus, agent i is not better off by deviating. If agent n takes action R and picks agent i, then it can be shown that the expected contribution of agent i is $\bar{x}_i$. Agent i is not better off by deviating. Thus, f is a subgame perfect equilibrium of the game $\Gamma(N,c)$. *Q.E.D.*

The next result says that the subgame perfect equilibrium outcome of the game $\Gamma(N,c)$ is unique. Moreover, it is $CEB(N,c)$.

Theorem 6: Each subgame perfect equilibrium outcome of the game $\Gamma(N,c)$ is $CEB(N,c)$.

Proof. Let g be a strategy profile that is a subgame perfect equilibrium of the game $\Gamma(N,c)$. Assume that each agent $k \neq n$ proposes x_k in Stage 1 by following g_k. Let $x_n = c_n - \sum_{k\neq n} x_k$ and $x = (x_k)_{k\in N}$. In stage 2, by following g_n, agent n could either take action A or take action R.

Case 1: Agent n takes action A by following g_n. Then, the game ends with outcome x. We first show that $x \in X(N,c)$, and then make use of Lemma 2 to conclude that $x = CEB(N,c)$. Suppose, by contradiction, that $x \notin X(N,c)$. Note that $\sum_{i\in N} x_i = c_n$ and for each $k \neq n$, by following g_k,

$0 \leq x_k \leq c_k$. Thus, either $x_n < 0$ or $x_n > c_n$. Suppose that $x_n > c_n$. Since $\sum_{k \in N} x_k = c_n$, there must be $k \in N \backslash \{n\}$ such that $x_k < 0$. This violates that for each $k \in N \backslash \{n\}$, $x_k \geq 0$. It follows that $x_n < 0$. Thus, there exists $i \in N \backslash \{n\}$ such that $x_i > 0$ and $c_n - \sum_{k \neq i,n} x_k < x_i$. We now claim that agent i will be better off by deviating. Let agent i deviate by proposing 0 in Stage 1. Let x' be the binding proposal induced by agent i's deviation. If in stage 2, agent n takes action A, then agent i's contribution is 0. Thus, agent i is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N, c)$. If in Stage 2, agent n takes action R and picks agent $k \in N \backslash \{i, n\}$, then agent i's contribution is 0. Thus, agent i is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N, c)$. If in Stage 2, agent n takes action R and picks agent i, then we consider two subcases.

Subcase 1.1: $\boldsymbol{c_n - \sum_{k \neq i,n} x_k < 0}$. Note that $x'_i = 0$, $x'_n = x_i + x_n$, and for each $k \in N \backslash \{i, n\}$, $x'_k = x_k$. Thus, the expected contribution of agent i is

$$
\begin{aligned}
\tau_i^i(c, x') &= \frac{1}{2} \left\{ \max\{0, c_i - \sum_{k \neq i,n} x'_k\} + (x'_i + x'_n) - \max\{0, c_n - \sum_{k \neq i,n} x'_k\} \right\} \\
&= \frac{1}{2} \{x_i + x_n\}.
\end{aligned}
$$

Note that $x_n < 0 < x_i$. Thus, $\tau_i^i(c, x') < x_i$. It follows that agent i is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N, c)$.

Subcase 1.2: $\boldsymbol{c_n - \sum_{k \neq i,n} x_k \geq 0}$. Note that $c_i \leq c_n$ and $c_n - \sum_{k \neq i,n} x_k < x_i$. Thus, $c_i - \sum_{k \neq i,n} x_k < x_i$. It follows that the expected contribution of agent i is

$$
\begin{aligned}
\tau_i^i(c, x') &= \frac{1}{2} \left\{ \max\{0, c_i - \sum_{k \neq i,n} x'_k\} + (x'_i + x'_n) - \max\{0, c_n - \sum_{k \neq i,n} x'_k\} \right\} \\
&= \frac{1}{2} \left\{ \max\{0, c_i - \sum_{k \neq i,n} x_k\} + (x_i + x_n) - \max\{0, c_n - \sum_{k \neq i,n} x_k\} \right\} \\
&< \frac{1}{2} \left\{ x_i + (x_i + x_n) - c_n + \sum_{k \neq i,n} x_k \right\} \\
&< x_i.
\end{aligned}
$$

Thus, agent i is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of $\Gamma(N, c)$. We conclude that $x \in X(N, c)$.

We next show that $x = CEB(N, c)$. Suppose, by contradiction, that $x \neq CEB(N, c)$. Then, by *converse consistency*, there exists $i \neq n$ such that $x_{\{i,n\}} \neq CEB\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. Note that $\left(uc^x_{\{i,n\}}\right)_n = c_n - \sum_{k \neq i,n} x_k = x_i + x_n$. There are two possibilities: either $CEB_n\left(\{i, n\}, uc^x_{\{i,n\}}\right) < x_n$ or $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right) < x_i$. Suppose that $CEB_n\left(\{i, n\}, uc^x_{\{i,n\}}\right) < x_n$. In this case, agent n has incentive to deviate in Stage 2 by taking action R and picking agent i, and since $x \in X(N, c)$, by Lemma 2, agent n ends up with the expected contribution $CEB_n\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. Thus, agent n is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N, c)$. Suppose now that $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right) < x_i$. In this case, agent i has incentive to deviate in Stage 1 by proposing $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. We next show that agent i will end up with contribution $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. If in Stage 2, agent n takes action A, then agent i ends up with contribution $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. If in Stage 2, agent n takes action R and picks agent $k \in N \backslash \{i, n\}$, then agent i ends up with contribution $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. If in Stage 2, agent n takes action R and picks agent i, then let x' be the binding proposal induced by agent i's deviation. It can be shown that $\tau^i_i(c, x') = \tau^i_i(c, x)$. Since $x \in X(N, c)$, by Lemma 2, $\tau^i_i(c, x) = CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. Thus, $\tau^i_i(c, x') = CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. It follows that agent i still ends up with contribution $CEB_i\left(\{i, n\}, uc^x_{\{i,n\}}\right)$. We conclude that in either case, agent i is better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N, c)$. Thus, $x = CEB(N, c)$.

Case 2: Agent n takes action R by following g_n. Assume that agent n takes action R and picks an agent, say agent i, in Stage 2. Let $y = \left(\tau^i(c, x), x_{N \backslash \{i,n\}}\right)$ be the outcome of the game. We show that $y = CEB(N, c)$. If $|N| = 2$, then $y = \tau^i(c, x) = (\frac{1}{2}c_i, c_n - \frac{1}{2}c_i) = CEB(N, c)$. We are done. Let $|N| \geq 3$. We first show that $x \in X(N, c)$. If x is not in $X(N, c)$, then there exists $k \in N\backslash\{n\}$ such that $x_k > 0$. If $k \in N\backslash\{i,n\}$, then agent k will deviate by proposing 0 in Stage 1 as agent i does in Case 1. We then derive that agent k will be better off by deviating, which violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$. If $k=i$, then agent n will be better off by taking action A and ending up with contribution x_n. To see this, we consider two situations: either $c_n - \sum_{j \neq k,n} x_j < 0$ or $c_n - \sum_{j \neq k,n} x_j \geq 0$. If $c_n - \sum_{j \neq k,n} x_j < 0$, then $\tau_k^k(c,x) = \frac{1}{2}(x_k + x_n)$. It follows that $y_n = \tau_n^k(c,x) = x_k + x_n - \tau_k^k(c,x) = \frac{1}{2}(x_k + x_n)$. Note that $x_n < 0 < x_k$. Thus, $y_n > x_n$. It follows that agent n is better off by deviating. If $c_n - \sum_{j \neq k,n} x_j \geq 0$, then it can be shown, as in Subcase 1.2, that $\tau_k^k(c,x) < x_k$. Thus, $y_n = \tau_n^k(c,x) = x_k + x_n - \tau_k^k(c,x) > x_n$. Agent n is better of by deviating. Thus, in either case, the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$ is violated. We next show that $y = CEB(N,c)$. Consider the following two subcases.

Subcase 2.1: For each $j \in N\backslash\{i,n\}$, $x_j = 0$. Then, $y_{\{i,n\}} = \tau^i(c,x) = \left(\frac{1}{2}c_i, c_n - \frac{1}{2}c_i\right)$. We now consider the following two situations.

- **For each $j \in N\backslash\{i,n\}$, $c_j \leq \frac{c_i}{2}$.** To solve $\sum_{k\in N} CEB_k(N,c) = c_n$, $\beta \in \mathbb{R}_+$ in the definition of the CEB rule must be $\frac{c_i}{2}$. Thus, $CEB(N,c) = \left(\frac{1}{2}c_i, c_n - \frac{1}{2}c_i, 0_{N\backslash\{i,n\}}\right)$. It follows that $y = CEB(N,c)$.

- **For some $j \in N\backslash\{i,n\}$, $c_j > \frac{c_i}{2}$.** We show that this situation never happens in the equilibrium. Consider the case when $c_i = 0$. Since $y_{\{i,n\}} = \left(\frac{1}{2}c_i, c_n - \frac{1}{2}c_i\right)$. Thus, $y_n = c_n$. Note that $i \neq n$, $c_i = 0$, and $0 \leq x_i \leq c_i$. Thus, $x_i = 0$. It follows that $x_n = c_n$ and for each $k \in N\backslash\{n\}$, $x_k = 0$. Thus,

$$\begin{aligned}\tau_j^j(c,x) &= \frac{1}{2}\left\{\max\{0, c_j - \sum_{k\neq j,n} x_k\} + x_j + x_n - \max\{0, c_n - \sum_{k\neq j,n} x_k\}\right\} \\ &= \frac{1}{2}(c_j + c_n - c_n) \\ &= \frac{c_j}{2}.\end{aligned}$$

  Agent n can deviate by picking agent $j \neq i$ in Stage 2 and obtain the expected contribution $\tau_n^j(c,x) = x_j + x_n - \tau_j^j(c,x) = c_n - \frac{1}{2}c_j$, which is less than c_n. Agent n is better off by deviating. It violates the supposition that g is a subgame perfect equilibrium of the game

$\Gamma(N,c)$. We next consider the case when $c_i > 0$. Then agent i can deviate by proposing a real number $\frac{1}{2}c_i - \varepsilon$, where

$$0 < \varepsilon < \min\{c_j - \frac{c_i}{2}, \frac{c_i}{2}\}.$$

Let x' be the binding proposal induced by agent i's deviation. Note that $x'_i = \frac{c_i}{2} - \varepsilon$, $x'_n = c_n - \frac{c_i}{2} + \varepsilon$, and for each $j \in N \backslash \{i,n\}$, $x'_j = x_j = 0$. Suppose that agent n picks agent j in Stage 2. Note that $y_n = c_n$ and $\tau^j_j(c,x') = \frac{1}{2}(c_j - \frac{1}{2}c_i + \epsilon)$. Then, agent n obtains the following expected contribution

$$\begin{aligned}
\tau^j_n(c,x') &= x'_j + x'_n - \tau^j_j(c,x') \\
&= c_n - \frac{1}{2}\left(c_j + \frac{1}{2}c_i - \varepsilon\right) \\
&= y_n - \frac{1}{2}\left(c_j + \frac{1}{2}c_i - \varepsilon\right) \\
&< y_n.
\end{aligned}$$

Thus, agent n will pick agent j, rather than agent i, in Stage 2. In this case, agent i will end up with contribution $\frac{1}{2}c_i - \varepsilon$, which is less than $\frac{c_i}{2}$. Thus, agent i will be better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$.

Subcase 2.2: For some $\boldsymbol{j \in N \backslash \{i,n\}}$, $\boldsymbol{x_j > 0}$. Since y is the outcome of the game $\Gamma(N,c)$, by subgame perfection, for each $k \in N\backslash\{i,n\}$, $y_n \leq \tau^k_n(c,x)$. We then consider two possible situations.

- **For each $\boldsymbol{k \in N\backslash\{i,n\}}$, $\boldsymbol{y_n < \tau^k_n(c,x)}$.** We show that this situation never happens in the equilibrium. Let $0 < \varepsilon < \min\{\tau^j_n(c,x) - y_n, x_j\}$. Suppose that agent j deviates by proposing $x_j - \varepsilon$. Let x' be the binding proposal induced by agent j's deviation. Since $x \in X(N,c)$, then $\max\{0, c_n - \sum_{k\neq i,n} x_k\} = x_i + x_n \geq 0$. Thus,

$$\begin{aligned}
\tau^i_i(c,x) &= \frac{1}{2}\left\{\max\{0, c_i - \sum_{k\neq i,n} x_k\} + x_i + x_n - \max\{0, c_n - \sum_{k\neq i,n} x_k\}\right\} \\
&= \frac{1}{2}\max\{0, c_i - \sum_{k\neq i,n} x_k\}.
\end{aligned}$$

Note that $y_n = \tau_n^i(c,x) = x_i + x_n - \tau_i^i(c,x)$. Thus, $y_n = x_i + x_n - \frac{1}{2}\max\{0, c_i - \sum_{k\neq i,n} x_k\}$. Note that $x \in X(N,c)$, $x'_j = x_j - \varepsilon$, $x'_n = x_n + \varepsilon$, and for each $k \in N\backslash\{j,n\}$, $x'_k = x_k$. Clearly, $x' \in X(N,c)$. Thus, $\tau_i^i(c,x') = \frac{1}{2}\max\{0, c_i - \sum_{k\neq i,n} x'_k\}$. It follows that

$$\begin{aligned}\tau_n^i(c,x') &= x_i + x_n + \varepsilon - \frac{1}{2}\max\left\{0, c_i - \sum_{k\neq i,n} x_k + \varepsilon\right\} \\ &\leq y_n + \varepsilon \\ &< \min\left\{\tau_n^j(c,x), x_j + y_n\right\} \\ &\leq \tau_n^j(c,x).\end{aligned}$$

Then, agent n will not pick agent j in Stage 2. Thus, agent j will end up with contribution $x_j - \varepsilon$, which is less than x_j. Agent j will be better off by deviating. This violates the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$.

- **For some $\boldsymbol{k \in N\backslash\{i,n\}}$, $\boldsymbol{y_n = \tau_n^k(c,x)}$.** If $y_{\{i,n\}} = x_{\{i,n\}}$, then $y = x$. We first show that $x = CEB(N,c)$, and then conclude that $y = CEB(N,c)$. Suppose, by contradiction, that $x \neq CEB(N,c)$. Then, by *converse consistency* of the CEB rule, there is $k \neq n$ such that $x_{\{k,n\}} \neq CEB\left(\{k,n\}, uc^x_{\{k,n\}}\right)$. Thus, either $x_n > CEB_n\left(\{k,n\}, uc^x_{\{k,n\}}\right)$ or $x_k > CEB_k\left(\{k,n\}, uc^x_{\{k,n\}}\right)$. If $x_n > CEB_n\left(\{k,n\}, uc^x_{\{k,n\}}\right)$, then in Stage 2, agent n will deviate by picking agent k and end up with contribution $CEB_n\left(\{k,n\}, uc^x_{\{k,n\}}\right)$. Thus, agent n is better off by deviating. If $x_k > CEB_k\left(\{k,n\}, uc^x_{\{k,n\}}\right)$, then in Stage 1, agent k will deviate by proposing $CEB_k\left(\{k,n\}, uc^x_{\{k,n\}}\right)$. By a similar argument as that for when $x_i > CEB_i\left(\{i,n\}, uc^x_{\{i,n\}}\right)$ in Case 1, it can be shown that agent k will end up with contribution $CEB_k\left(\{k,n\}, uc^x_{\{k,n\}}\right)$. Thus, agent k is better off by deviating. It follows that in either case, the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$ is violated. Thus, $x = CEB(N,c)$. Now, let's consider when $y_{\{i,n\}} \neq x_{\{i,n\}}$. We show that agent i will be better off by deviating, and thus the supposition that g is a subgame perfect equilibrium of the game $\Gamma(N,c)$ is violated. Since y is the outcome of the game $\Gamma(N,c)$, by subgame perfection, $y_n < x_n$. Note that $\sum_{i\in N} x_i = \sum_{i\in N} y_i$ and $y_{N\setminus\{i,n\}} = x_{N\setminus\{i,n\}}$. Thus, $x_i < y_i$. Since $x \in X(N,c)$, by Lemma 2, $y = \tau^i(N,c) = CEB\left(\{i,n\}, uc^x_{\{i,n\}}\right)$. Note that by *order preservation for contributions* of the CEB rule, $0 < y_i \leq y_n$. Suppose that agent i deviates by proposing $x_i + \varepsilon$, where $0 < \varepsilon < \min\{y_i - x_i, x_n\}$. Let x' be the binding proposal induced by agent i's deviation. Note that $x'_i = x_i + \varepsilon$, $x'_n = x_n - \varepsilon$, and for each $l \in N\setminus\{i,n\}$, $x'_l = x_l$. Thus, $x' \in X(N,c)$. Thus, $\tau^k_k(c,x') = \frac{1}{2}\max\{0, c_k - \sum_{l\neq k,n} x'_l\}$. It follows that

$$\begin{aligned}
\tau^k_n(c,x') &= x_k + x_n - \varepsilon - \frac{1}{2}\max\left\{0, c_k - \sum_{l\neq k,n} x_l - \varepsilon\right\} \\
&= x_k + x_n - \frac{1}{2}\max\left\{2\varepsilon, c_k - \sum_{l\neq k,n} x_l + \varepsilon\right\} \\
&< x_k + x_n - \frac{1}{2}\max\left\{0, c_k - \sum_{l\neq k,n} x_l\right\} \\
&= \tau^k_n(c,x) \\
&= y_n \\
&= \tau^i_n(c,x)
\end{aligned}$$

Agent n will not pick agent i in Stage 2. It follows that agent i will end up with contribution $x_i + \varepsilon$, which is less than y_i. Agent i is then better off by deviating.

By Subcases 2.1 and 2.2, we obtain that $y = CEB(N,c)$. Thus, by Cases 1 and 2, we complete the proof. *Q.E.D.*

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