On the Measurement of Long-Term Income Inequality and Income Mobility

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Abstract

This paper proposes a two-step aggregation method for measuring long-term income inequality and income mobility, where mobility is defined as an equalizer of long-term income. The first step consists of aggregating the income stream of each individual into a measure of permanent income, which accounts for the costs associated with income fluctuations and allows for credit market imperfections. The second step aggregates permanent incomes across individuals into measures of social welfare, inequality and mobility. To this end, we employ an axiomatic approach to justify the introduction of a generalized family of rank-dependent measures of inequality, where the distributional weights, as opposed to the Mehran-Yaari family, depend on income shares as well as on population shares. Moreover, a subfamily is shown to be associated with social welfare functions that have intuitively appealing interpretations. Further, the generalized family of inequality measures provides new interpretations of the Gini-coefficient.

Keywords: Income inequality, income mobility, social welfare, the Gini coefficient, permanent income, credit market, annuity.

JEL classification: D71, D91, I32

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1. Introduction

More than half a century ago, Friedman (1962) suggested that a proper understanding of income inequality requires taking income mobility into account. The line of reasoning was that high annual income inequality might occur side by side with little or no inequality in long-term incomes, if individuals' positions in the annual income distributions change over time. This motivated a considerable theoretical and empirical literature, starting with Shorrocks (1978), where mobility is defined as an equaliser of long-term or permanent incomes. This notion of mobility is measured as the change in income inequality when extending the accounting period of income, and requires aggregation in two steps.[1] The first step consists of aggregating the income stream of each individual into an interpersonal comparable measure of permanent income, whereas the second step deals with the problem of aggregating the individual permanent incomes into measures of social welfare, inequality, and mobility. The purpose of this paper is to introduce a framework for measuring income mobility that contributes to the existing literature in both regards.

In Shorrocks (1978) as well as in most subsequent empirical studies of income mobility, the average (real) income over several years is used as an approximation for permanent income.[2] This means that the two-period income streams (50, 100) and (75, 75) will be considered to produce the same level of permanent income. Accordingly, this approach pays no attention to the fact that mobility may imply income instability for the individuals which will matter for their welfare if it is costly to transfer income between time periods. In fact, this problem was acknowledged by Shorrocks (1978), and is a common criticism of studies of mobility as an equaliser of long-term average incomes (see e.g. Chakravarty et al., 1985; Aktinson et. al., 1992 and Fields and Ok, 1999).

To develop a method for measuring mobility where high mobility, everything else equal, is socially preferable, it is necessary to introduce a measure of permanent income that incorporates the costs of and constraints on making inter-period income transfers. To this end, we draw on the intertemporal choice theory and define permanent income as *the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers.* The minimum annual expenditure will be denoted the equally allocated equivalent income. To justify interpersonal comparison of the equally allocated equivalent incomes, we follow standard practice in assuming that the inter-period income transfers are carried out in accordance with an intertemporal

[1] We refer to Chakravarty et al. (1985), Atkinson et al. (1992), Dardanoni (1993), Fields and Ok (1996, 1999), Fields (2009), Gottschalk and Spolare (2002), Tsui (2009) and D'Agostino and Dardanoni (2009) for discussions of alternative definitions and methods for measuring mobility.

[2] A number of studies measure long-term inequality and mobility based on average income, including Shorrocks (1978), Chakravarty et al. (1985), Bjørklund (1993), Burkhauser and Puopore (1997), Maasoumi and Trede (2001), Aaberge et al. (2002), Ruiz-Castello (2004), and Ayala and Sastre (2004).

utility function that is common to all individuals. The common intertemporal utility function is to be determined by the social planner, and can be viewed as a normative standard where individuals are treated symmetrically.[3]

Provided that the instantaneous utility term of the intertemporal utility function belongs to the much used Bergson family, our permanent income measure proves to be equal to the utility-equivalent annuity introduced by Nordhaus (1973). Nordhaus (1973) and Creedy (1999) express, however, concern about the sensitivity of distributional analysis based on the utility-equivalent annuity measure to the choice of preference parameters. As will be demonstrated below, their concern is uncalled for because the utility-equivalent annuity proves to be the product of two terms; one that is a function of the income stream and another that is a function of the preference parameters. Since the latter term is common to all individuals, measures of inequality and mobility will (due to scale-invariance) solely depend on the income stream term. This result provides a theoretical underpinning to using the annuity of an individual's income stream as a measure of permanent income in studies of long-term inequality and mobility.

While the first aggregation step maps the income stream of each individual into a measure of permanent income, the purpose of the second step is to aggregate permanent incomes across individuals into measures of long-term income inequality, social welfare and income mobility, when the state of immobility is defined as no changes over time in individuals' ranks in the short term distributions of income. This calls for measures of mobility that are derived from rank-dependent measures of inequality of the type introduced by Mehran (1976) and Yaari (1988). The Mehran-Yaari family of rank-dependent measures of inequality can be considered as a weighted sum of income shares where the weights depend on population shares but not on the income shares. To illustrate the shortcoming of these inequality measures, consider a population divided into a group of poor and a group of rich, where each individual's income is equal to the corresponding group mean. Applying the Mehran-Yaari family of inequality measures, the inequality reduction of an income transfer from the rich to the poor will depend solely on relative number of poor people, irrespective of their share of total income. To account for the impact of population shares as well as income shares, we introduce a more general family of rank-dependent measures of inequality which is justified to represent an ordering relation on the set of Lorenz curves. Due to their convenient expressions, it is straightforward to estimate these inequality measures, which shows to supplement each other with regard to sensitivity to changes in the lower, the central and the upper part of the income distribution. A subfamily of this generalized family of rank-dependent measures of inequality is shown to be

[3] The use of a common utility function is well-established in the public economic literature and has e.g. been proposed by Deaton and Muellbauer (1980) and Hammond (1991). It also forms the basis for the definition and measurement of a money-metric measure of utility in for example King (1983) and Aaberge et al. (2004).

associated with social welfare functions that prove to have intuitively appealing interpretations. Further, the generalized family of rank-dependent inequality measures provides new interpretations of the Gini coefficient.

Finally, we introduce a new family of rank-dependent measures of income mobility that rely on *(i)* the introduced measure of permanent income, and *(ii)* the general family of rank-dependent measures of income inequality. On this basis, income mobility is defined as the reduction in inequality in the distribution of permanent income due to changes over time in individuals' ranks and income shares in the short term distributions of income. Mobility will have an unambiguously positive impact on social welfare in the sense that if two societies have identical short term income distributions, then social welfare will be greatest for the society which exhibits most mobility. Further, the proposed family of rank-dependent measures of income mobility proves to encompass standard measures of income mobility, depending on the assumptions made by the social planner about the intertemporal preferences of individuals and the credit market.

It should be noted that it is straightforward to use our method to measure income mobility when the distribution of income in the first year forms the benchmark distribution, as has been proposed by e.g. Chakravarty et al. (1985), Benabou and Ok (2001), Ruiz-Castillo (2004), and Fields (2009). In this way, the mobility measures convey how inequality of permanent incomes compares with the inequality of first-year incomes.

Section 2 proceeds by describing the method for aggregating the income streams of individuals into comparable measures of permanent income. Section 3 deals with the problem of aggregating permanent incomes across individuals into measures of long-term inequality and social welfare. Section 4 introduces a new family of rank-dependent measures of income mobility, whereas Section 5 summarizes the main results of the paper and relates them to alternative approaches to measuring long-term inequality and income mobility.

2. Definition and measurement of permanent income

Below, we propose a method for measuring permanent income that conforms to the basic structure of intertemporal choice theory, and justifies comparison of permanent incomes across individuals. First, we consider the case of a perfect credit market, before extending the method to account for credit market imperfections.

2.1. Perfect credit market

2.1.1. Equally allocated equivalent income

In analysis of long-term income inequality and mobility, the problem of interpersonal comparability of income streams arises. To justify interpersonal comparison of individuals' income streams, we follow standard practice in assuming that inter-period income transfers are carried out in accordance with an intertemporal utility function that is common to all individuals. The common utility function is to be determined by the social planner based on his ethical value judgement, and contains within it interpersonal comparability of both welfare levels and welfare differences. Rather than claiming that the common utility function is a descriptively accurate representation of the behaviour of heterogeneous individuals, it is justified as a normative standard where the social planner treats individuals symmetrically after adjusting for relevant non-income heterogeneity, such as employing equivalence scales to adjust for household size and composition. Specifically, our permanent income measure is defined as

> *the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers according to a common intertemporal utility function subject to his budget constraints*.

To provide a formal counterpart to this definition, the social planner is assumed to employ the conventional discounted utility model with perfect foresight, where preferences are intertemporal separable and additive.[4] The instantaneous common utility function u is assumed to be stationary, increasing, concave, and differentiable. Furthermore, we assume that the rate of time preference δ is non-negative and constant over time. Let $(C_1, C_2, .., C_T)$ and $(Y_1, Y_2, .., Y_T)$ be an individual's stream of consumption levels and exogenous real disposable incomes net of interests for an individual. Under the assumption of a perfect credit market, the real interest rates on savings and borrowing are equal across the population, though they may vary over time. Let r_t denote the real interest rates on income-transfers from period $t-1$ to t. From the viewpoint of the social planner, the individual's preferred consumption profile $(C_1^*, C_2^*, ..., C_T^*)$ is defined as the solution of

$$\text{(2.1)} \qquad \max_{C_1, C_2, \ldots, C_T} \sum_{t=1}^{T} u(C_t)(1+\delta)^{1-t}$$

[4] See Koopmans (1960) for an attempt to axiomatically justify the discounted utility model in general, and Kahneman et al. (1997) for an axiomatic rationalisation of the assumption of additive separability in instantaneous utility. As is well known, the discounted utility model can straightforwardly be extended to allow for uncertainty.

subject to the budget constraint[5]

(2.2) $$\sum_{t=1}^{T} C_t \prod_{j=1+t}^{T}(1+r_j) + C_T = \sum_{t=1}^{T} Y_t \prod_{j=1+t}^{T}(1+r_j) + Y_T .$$

As is well known, the preferred consumption level in period *t*, C_t^*, can be expressed as a function of the preferred consumption level in period 1

(2.3) $$u'(C_t^*) = \frac{(1+\delta)^{t-1}}{\prod_{j=2}^{t}(1+r_j)} u'(C_1^*), \quad t = 2,3,...,T.$$

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From (2.3) and (2.2), C_t^* can be expressed as a function f_t of δ, $Y_1, Y_2,.., Y_T$, and $r_2, r_3,.., r_T$

(2.4) $$C_t^* = f_t(\delta, Y_1, Y_2,..., Y_T, r_2, r_3,..., r_T) \text{ for all } t=1, 2, \ldots, T.$$

Inter-period income transfers are carried out to ensure that the marginal utility of consumption is constant over time, which generally will result in preferred consumption levels that differ between time periods. By inserting for (2.4) in (2.1) the maximum utility level (U) is given by

(2.5) $$U = \sum_{t=1}^{T} u(C_t^*)(1+\delta)^{1-t}$$

As suggested above the minimum annual expenditure (Z) that an individual requires to obtain the maximum utility level U emerges as an appropriate representation of permanent income. Replacing the preferred C_t^* with Z for every t in the second term of equation (2.5) yields

(2.6) $$Z = u^{-1}(\Delta^{-1}U)$$

where $u^{-1}(t) = \inf\{x : u(x) \geq t\}$ is the left inverse of u and Δ is defined by

(2.7) $$\Delta = \sum_{t=1}^{T}(1+\delta)^{1-t} = \frac{1+\delta}{\delta}\left(1-(1+\delta)^{-T}\right).$$

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[5] It is straightforward to extent the budget constraint to account for wealth, e.g. by assuming that the income in the first period Y_1 in (2.2) includes the initial stock of wealth.

The minimum annual expenditure Z will be denoted the equally allocated equivalent income (EAEI). Since the individual-specific EAEI can be considered to be interpersonal comparable money-metric measures of the maximum utility levels, the distribution of EAEI may form the basis for studying long-term inequality and income mobility.

Note that the EAEI can be considered as an analogous to the certainty equivalent in the theory of choice under uncertainty and the equally distributed equivalent income in analyses of income inequality (see Atkinson, 1970). While the equally distributed equivalent income represents a money-metric measure of the social welfare for a given distribution of income across individuals, the EAEI represents a money-metric measure of the well-being level associated with the income stream for a given individual. Thus, the social planner considers the income stream $(Y_{i1}, Y_{i2},.., Y_{iT})$ of individual i to be preferable to the income stream $(Y_{j1}, Y_{j2},.., Y_{jT})$ of individual j if and only if Z_i exceeds Z_j.

2.1.2. Annuity as a measure of permanent income

A benchmark case in intertemporal choice theory uses the annuity (A) of the income stream as measure of permanent income (see e.g. Meghir, 2004). The annuity income is defined by

(2.8)
$$A = \frac{Y_T + \sum_{t=1}^{T-1} Y_t \prod_{j=1+t}^{T} (1 + r_j)}{1 + \sum_{t=1}^{T-1} \prod_{j=1+t}^{T} (1 + r_j)},$$

when T is the basis for the annuity calculations. When $r_2 = r_3 = \cdots = r_T = \delta$, it follows directly that the EAEI coincides with the annuity income,

(2.9)
$$Z = A = \frac{\sum_{t=1}^{T} (1+\delta)^{T-t} Y_t}{\sum_{t=1}^{T} (1+\delta)^{T-t}}.$$

Thus, it is clear that A is an appropriate measure of permanent income insofar it is reasonable for the social planner to assume that the real interest rates are constant over time and equal to the rate of time preferences. The behavioural counterpart is that individuals' prefer to carry out equalizing income transfers to achieve a constant consumption level over time. Hence, it is apparently required to impose rather restrictive conditions to justify the use of the annuity as a measure of permanent income in analysis of long-term income inequality and income mobility.

An interesting question is whether A defined by (2.8) remains valid as a measure of comparable permanent income in analysis of long-term inequality and mobility even in cases where the condition of constant consumption levels over time is abandoned. To address this question, we replace the assumption of $r_2 = r_3 = \cdots = r_T = \delta$ with the less restrictive assumption of consumption proportionality,

(2.10) $$C_t^* = q_t C_1^*, \; t=1, 2, 3,\ldots, T,$$

where q_t is defined implicitly by

(2.11) $$g(q_t) = \frac{(1+\delta)^{t-1}}{\prod_{j=2}^{t}(1+r_j)}, \; t=2, 3, \ldots, T,$$

$g(x) = u'(x)$ and $q_1 = 1$. In this case, the ratio between the optimal consumption levels for two arbitrarily chosen periods depends on the instantaneous utility function u, the rate of time preference δ, and the real interest rates $(r_2, r_3, \ldots, r_T)$ but not on the income stream $(Y_1, Y_2, \ldots, Y_T)$. As demonstrated by Theorem 2.1 below, the consumption profile (2.10) is optimal if and only if the utility function is a member of the Bergson family, which is a much used specification of the instantaneous utility function in intertemporal choice theory (Davies and Shorrocks, 2000).

Theorem 2.1. *Let $(C_1^*, C_2^*, \ldots, C_T^*)$ be the vector of optimal consumption levels for periods 1,2,...T defined by (2.3) where u' is the derivative of the instantaneous utility function u, and let q_t be defined by (2.11). Then*

(i) $$C_t^* = q_t C_1^* \quad \text{for } t = 1,2,3,\ldots,T$$

if and only if

the instantaneous utility function u is a member of the Bergson family

(ii) $$u(x) = \begin{cases} \dfrac{1}{1-\varepsilon}(x^{1-\varepsilon} - 1) & \text{if } \varepsilon \neq 1 \\ \log x & \text{if } \varepsilon = 1, \end{cases}$$

where ε^{-1} is the intertemporal elasticity of substitution.

Proof. Assume that $C_t^* = q_t C_1^*$ where q_t is defined by (2.11). By inserting for (2.10) and (2.11) in (2.3) we obtain the following functional equation

$$g(q_t C_1^*) = g(q_t) g(C_1^*) \quad \text{for all } q_t \text{ and } C_1^*,$$

which has the solution (see Aczél, 1966) $g \equiv 0$ or 1, or there exists a real number ε^{-1} such that

$$u'(x) = g(x) = x^{-\varepsilon}.$$

Hence, (i) implies (ii).
The converse statement follows by inserting (ii) in (2.3). □

Remark. *The result given in Theorem 2.1 is analogous to a consumer behaviour result of Burk (1936), where it is demonstrated that demand functions exhibit expenditure proportionality if and only if the utility function belongs to the Bergson family (ii). However, whilst the proof above is rather simple and solely requires the solution of a well-known functional equation, the proof given by Burk is more complex and requires the solution of a set of differential equations.*[6]

As will be demonstrated below, the result of Theorem 2.1 proves useful for identifying the relationship between A defined by (2.8) and Z defined by (2.6). To this end, it is convenient to introduce the notation a_t defined by

(2.12)
$$a_t = \frac{1 + \sum_{t=1}^{T-1} \prod_{j=t+1}^{T} (1 + r_j)}{\sum_{t=1}^{T-1} q_t \prod_{j=t+1}^{T} (1 + r_j) + q_T} q_t, \quad t = 1,2,\ldots,T,$$

and $k(\varepsilon,\delta)$ defined by[7]

[6] See also Samuelson (1965). Moreover, Pratt (1964) demonstrates that an economic agent who acts in accordance with the criterion of expected utility when he makes decisions under risk exhibits constant relative risk aversion if and only if the utility function is a member of the Bergson family.

[7] For convenience the dependence of k on $r_2, r_3, \ldots, r_T$ is suppressed in the notation for k.

(2.13)
$$k(\varepsilon,\delta)=\begin{cases}\left[\dfrac{\sum_{t=1}^{T}a_t^{1-\varepsilon}(1+\delta)^{1-t}}{\Delta}\right]^{\frac{1}{1-\varepsilon}} & \text{when } \varepsilon\neq 1\\ \prod_{t=1}^{T}a_t^{\frac{(1+\delta)^{1-t}}{\Delta}} & \text{when } \varepsilon=1,\end{cases}$$

where Δ is defined by (2.7).

Theorem 2.2. *Let $(C_1^*,C_2^*,...,C_T^*)$ be the vector of optimal consumption levels for periods 1,2,...,T defined by (2.3) where u' is the derivative of the instantaneous utility function u, δ is the rate of time preferences and $r_2,r_3,...,r_T$ are the real interest rates. Moreover, let Z, A, q_t and $k(\varepsilon,\delta)$ be defined by (2.6), (2.8), (2.11) and (2.13). Then*

(i)
$$C_t^*=q_tC_1^* \quad \text{for } t=1,2,3,...,T$$

implies

(ii)
$$Z=k(\varepsilon,\delta)A.$$

Proof. By inserting for $C_t^*=q_tC_1^*$ in equation (2.2) we get

(2.14)
$$C_1^*=\frac{Y_T+\sum_{t=1}^{T-1}Y_t\prod_{j=t+1}^{T}\left(1+r_j\right)}{\sum_{t=1}^{T-1}q_t\prod_{j=t+1}^{T}(1+r_j)+q_T}.$$

Next, inserting for (2.8), (2.12) and (2.14) in $C_t^*=q_tC_1^*$ yields

(2.15)
$$C_t^*=a_tA \quad \text{for } t=1,2,3,...,T.$$

Moreover, when (2.15) is true then it follows from Theorem 2.1 that the instantaneous utility function u is given by (ii) of Theorem 2.1. By inserting (2.15) and specification (ii) of Theorem 2.1 for u in equation (2.5) we get

(2.16)
$$U=\begin{cases}\dfrac{1}{1-\varepsilon}\left[A^{1-\varepsilon}\sum_{t=1}^{T}a_t^{1-\varepsilon}(1+\delta)^{1-t}-\Delta\right] & when\ \varepsilon\neq 1\\ \Delta\log A+\sum_{t=1}^{T}(1+\delta)^{1-t}\log a_t & when\ \varepsilon=1.\end{cases}$$

Now, inserting for (2.16) and the inverse of the Bergson utility function *u* (defined by (ii) of Theorem 2.1) in (2.6) yields

(2.17)
$$Z=k(\varepsilon,\delta)A. \qquad \square$$

Note that the EAEI coincides with the utility-equivalent annuity measure introduced by Nordhaus (1973), provided that the instantaneous utility function is of the Bergson type. Nordhaus (1973) as well Creedy (1999) express concern about the sensitivity of the analysis of distributional analysis based on utility-equivalent annuity measures to the choice of values for ε and δ. However, it follows from Theorem 2.2 that scale-invariant measures of inequality based on the utility-equivalent annuity measure are independent of ε and δ, and solely depend on A. This result provides a theoretical underpinning to using the annuity of an individual's income stream as a measure of permanent income in studies of long-term inequality and mobility, even when the real interest rates differ from the rate of time preferences.

2.2. Credit marked imperfections

When interest rates on borrowing and savings differ then (2.2) is no longer a valid representation of the budget constraints. Consequently, the preferred consumption levels defined as the solution to (2.1) and (2.2) will in this case not form an appropriate basis for defining and measuring the EAEI.

Formally, we can apply the Kuhn-Tucker method to derive the preferred consumption profiles in the case of imperfect credit markets. For simplicity, assume that each individual is faced with a single borrowing interest rate and a single savings interest rate (but different individuals may face different interest rates on borrowing and/or savings). If there are no liquidity constraints, the preferred consumption profile ($C_1^*,C_2^*,...,C_T^*$) is defined as the solution of (2.1) subject to the budget constraints

(2.18)
$$\begin{aligned}&S_0=0\\&S_t=(1+r\gamma_t)S_{t-1}+Y_t-C_t,\\&S_T=(1+r\gamma_t)S_{T-1}+Y_T-C_T=0\end{aligned}$$

where S_t represents the assets at the end of period t earning an interest rate $r\gamma_{t+1}$, and

$$(2.19) \qquad r\gamma_t = \begin{cases} rs_t & if\ S_{t-1} \geq 0 \\ rb_t & if\ S_{t-1} < 0 \end{cases} ,-1 < r\gamma_t < \infty, \quad t = 2,3,...,T ,$$

where rs_t and rb_t denote the saving and borrowing rates. Solving this maximization problem requires comparison of 3^{T-1} conditional consumption profiles for each individual. The conditional consumption profiles are distinctive in terms of whether individual i in the various periods is considered to be a saver, a borrower, or locate at the kink and thereby consume all his assets. Each of these conditional consumption profiles is a candidate for the individual's preferred consumption profile provided that the budget constraints are satisfied for the given values of Y_t and $r\gamma_t$. The optimal consumption profile is determined as the utility maximising choice among the conditional consumption profiles satisfying the budget constraints. By inserting the consumption levels of the optimal consumption profiles into (2.5), the corresponding Z is obtained from (2.6).

Presence of liquidity constraints will reduce the number of available conditional consumption profiles that have to be compared. For example, the case where borrowing in each period is prohibited corresponds to reducing the number of conditional consumption profiles to those satisfying $S_t \geq 0$. Thus, deriving EAEI subject to liquidity constraints is straightforward and can be considered as a special case of the method outlined above.

3. Generalized rank-dependent measures of income inequality

This section discusses how to aggregate permanent incomes across individuals into measures of income inequality and social welfare, when the state of immobility is defined as no changes over time in individuals' ranks in the short term distributions of income. This calls for rank-dependent measures of inequality that can be justified to represent preference orderings over Lorenz curves. By displaying the deviation in each individual's income share from the income share that corresponds to perfect equality, the Lorenz curve captures the essential descriptive features of the concept of inequality. The normative aspect of ranking Lorenz curves will be discussed below.

3.1. Two alternative families of rank-dependent measures of inequality

In theories of choice under uncertainty, preference orderings over probability distributions are introduced as a basis for deriving utility indices. In a similar vain, appropriate preference relations on the set of Lorenz curves can be introduced to derive inequality indices.

The Lorenz curve L for a cumulative income distribution F with mean μ is defined by

$$L(u) = \frac{1}{\mu}\int_{F(x)\leq u} x dF(x), \tag{3.1}$$

where L is an increasing convex function with range [0,1]. Thus, L can be considered analogous to a convex distribution function on [0,1] and the problem of ranking Lorenz curves can, formally, be viewed as analogous to the problem of choice under uncertainty.

Let $\boldsymbol{L}$ denote the family of Lorenz curves, and let a social planner's ranking of members of $\boldsymbol{L}$ be represented by a preference ordering $\succeq$, which will be assumed to satisfy the following basic axioms:

Axiom 1 (Order). $\succeq$ *is a transitive and complete ordering on* $\boldsymbol{L}$*.*

Axiom 2 (Dominance). *Let* $L_1, L_2 \in \boldsymbol{L}$. *If* $L_1(u) \geq L_2(u)$ *for all* $u \in [0,1]$ *then* $L_1 \succeq L_2$.

Axiom 3 (Continuity). *For each* $L \in \boldsymbol{L}$, *the sets* $\left\{L^* \in \boldsymbol{L} : L \succeq L^*\right\}$ *and* $\left\{L^* \in \boldsymbol{L} : L^* \succeq L\right\}$ *are closed (w.r.t.* L_1*-norm).*

Given the above continuity and dominance assumptions for the ordering $\succeq$, Aaberge (2001) demonstrated that the following axiom,

Axiom 4 (Independence). *Let* L_1, L_2 *and* L_3 *be members of* $\boldsymbol{L}$ *and let* $\alpha \in [0,1]$. *Then* $L_1 \succeq L_2$ *implies* $\alpha L_1 + (1-\alpha)L_3 \succeq \alpha L_2 + (1-\alpha)L_3$, characterizes the rank-dependent family of inequality measures $\tilde{J}_p$ defined by[8]

$$(3.2) \qquad \tilde{J}_p(L) = 1 + \int_0^1 L(u)dp(u),$$

where L is the Lorenz curve and p is a positive and non-increasing function defined on the unit interval such that $\int udp(u) = -1$.[9] Note that p can be interpreted as a preference function of a social planner that assigns weights to the incomes of the individuals in accordance with their rank in the income distribution. Therefore, the functional form of p reveals the attitude towards inequality of a social planner who employs $\tilde{J}_p$ to judge between Lorenz curves.

The welfare economic justification for the family of rank-dependent measures of inequality $\tilde{J}_p$ is analogous to the justification for Atkinson's expected utility type of inequality measures. The essential differences between these two approaches for measuring inequality and social welfare arise from the independence axioms. Whilst the expected utility independence axiom requires that the ordering of distributions of individual welfare is invariant with respect to identical mixing of the distributions being compared, the rank-dependent independence axiom requires that the ordering is invariant with respect to identical mixing of Lorenz curves (or identical mixing of the inverses of distributions) being compared. For further discussion, see Yaari (1988) and Aaberge (2001).

As suggested above, the $\tilde{J}_p$-measures can be viewed as a sum of weighted income shares where the weights depend on population shares but not on the income shares. Aaberge (2001) shows that a family of inequality measures J_q^* with weights that depend on income shares – but not on population shares – are obtained by replacing Axiom 4 with

Axiom 5 (Dual independence). *Let L_1, L_2 and L_3 be members of* $\mathbf{L}$ *and let* $\alpha \in [0,1]$. *Then* $L_1 \succeq L_2$ *implies* $\left(\alpha L_1^{-1} + (1-\alpha)L_3^{-1}\right)^{-1} \succeq \left(\alpha L_2^{-1} + (1-\alpha)L_3^{-1}\right)^{-1}$.

The family J_q^* is defined by

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[8] Mehran (1976) introduced an alternative version of (3.2) based on descriptive arguments. For alternative motivations of the J_P-family and various subfamilies we refer to Donaldson and Weymark (1980, 1983), Weymark (1981), Yitzhaki (1983), Ben Porath and Gilboa (1994), and Aaberge (2000, 2007).

[9] Note that Yaari (1987, 1988) provides an axiomatic justification for using $\int_0^1 p(u)F^{-1}(u)du = \mu(1-\tilde{J}_p(L))$ as a criterion for ranking distribution functions F.

(3.3) $$J_q^*(L) = 1 - \int_0^1 q(L(u))du ,$$

where q is a positive and non-decreasing function defined on the unit interval such that $\int q(t)dt = 1$. As can be observed from (3.3), the weights of J_q^* depend on Lorenz curve values (income shares).

By choosing $p(u) = 2(1-u)$ for $\tilde{J}_p$ and $q(t) = 2t$ for J_q^* it follows directly from (3.2) and (3.3) that the Gini coefficient is a member of $\tilde{J}_p$ as well as of J_q^*. Note that the Gini coefficient is the only measure of inequality that is a member of both $\tilde{J}_p$ and J_q^*. Moreover, by choosing appropriate specifications for p and q we can derive alternatives to the Gini coefficient (see Aaberge, 2000, 2001).

3.2. A new general family of rank-dependent measures of inequality

Consider a population divided into a group of poor and a group of rich where each individual's income is equal to the corresponding group mean. In this case, the effect on $\tilde{J}_p$-measures of increasing the income share of the poor depends solely on the relative number of poor irrespective of their share of income, while a similar effect on J_q^*-measures depends both on the poor's share of the population and their incomes. By contrast, the effect on J_q^*-measures of an increase in the relative number of poor depend merely on the poor's share of the incomes, whereas the effect on $\tilde{J}_p$-measures depend on the proportion of poor as well as on their income share. Thus, it appears attractive to construct a family of inequality measures that combines the basic features of the families $\tilde{J}_p$ and J_q^*. To this end, it is required to introduce an axiom that can be considered as a generalization of Axiom 4 as well as of Axiom 5.

An intuitively appealing axiom introduced by Green and Jullien (1988) to resolve paradoxes in the theory of choice under uncertainty,[10] appears equally attractive for describing preferences for the ranking of Lorenz curves, not least since this axiom can be considered as a weakening of Axiom 4 as well as of Axiom 5.

Axiom 6. (Ordinal independence). *Let L_1, L_2, L_3 and L_4 be members of* $\mathbf{L}$ *and let* $a \in [0,1]$. *If for every* $u \leq a$ $L_1(u) = L_2(u)$ *and* $L_3(u) = L_4(u)$ *and for every* $u \geq a$ $L_1(u) = L_3(u)$ *and* $L_2(u) = L_4(u)$, *then* $L_1 \succeq L_2$ *if and only if* $L_3 \succeq L_4$

[10] For alternative axiomatizations of rank-dependent expected utility, see Quiggen (1982, 1989, 1993), Yaari (1987), Chew and Epstein (1989), Segal (1989, 1990), and Wakker (1994).

Figure 1. Illustration of the ordinal independence axiom

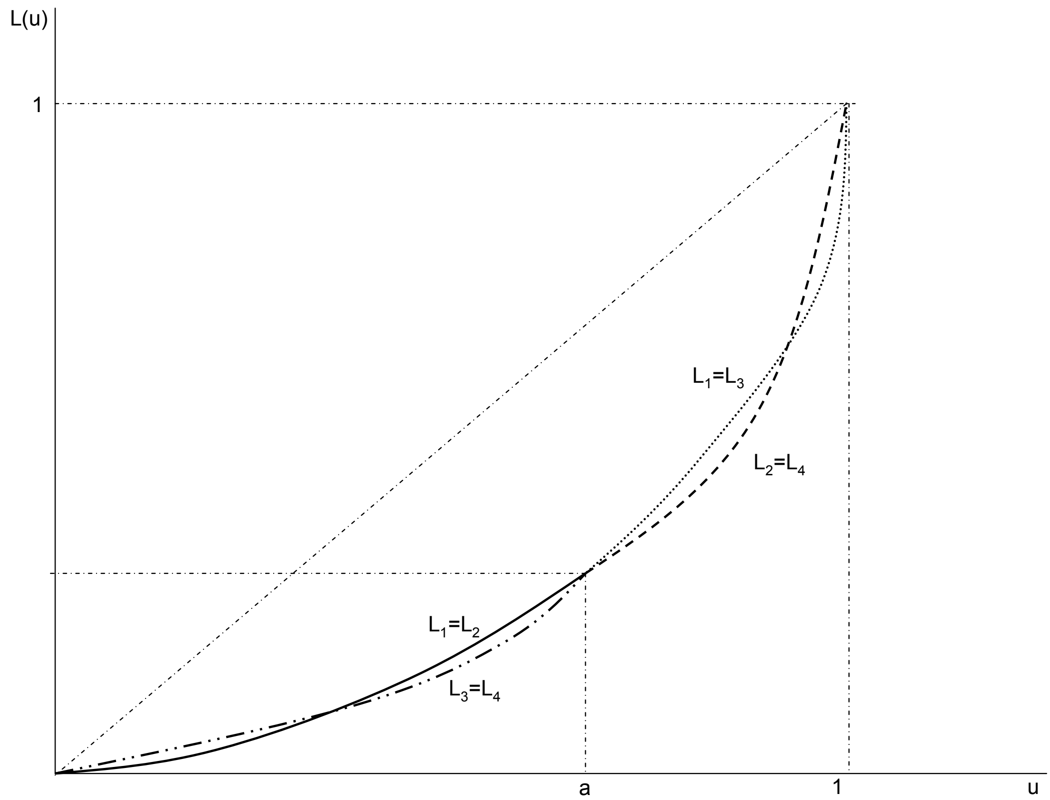

Figure 1 provides an illustration of the ordinal independence axiom, where $L_1(u) = L_2(u)$ and $L_3(u) = L_4(u)$ for $u \leq a$, while $L_1(u) = L_3(u)$ and $L_2(u) = L_4(u)$ for $u \geq a$. The ordinal independence axiom states that $L_1 \succeq L_2$ if and only if $L_3 \succeq L_4$. Thus, Axiom 6 asserts that preferences between two Lorenz curves with a common tail will be unaffected by the any changes of this common tail. To clarify the interpretation of the ordinal independence axiom, Figure 1 draws an example where two Lorenz curves L_1 and L_2 differ above an intersection point a and coincides below a. Assume that the preferences of a social planner is consistent with $L_1 \succeq L_2$. Now, consider a policy change that transfers income from the richest to the poorest among the 100a per cent poorest of the population of L_1 and L_2, such that L_1 equals L_3 and L_2 equals L_4, after the intervention. Then, Axiom 6 states that the changes in L_1 and L_2 that follow from this intervention will not affect the ranking of the Lorenz curves, irrespective of how incomes are distributed among the poorest 100a percent after the intervention. This implies that a social planner who is in favour of employing general criteria of upward (aggregation from below) or downward (aggregation from above) Lorenz dominance will always act in accordance with the ordinal independence axiom.[11]

Analogous to what Green and Jullien (1988) proved for rank-dependent expected utility, we get

[11] We refer to Aaberge (2009) for a definition of two separate (upward and downward) nested sequences of Lorenz dominance criteria.

Theorem 3.1. *A preference relation* $\succeq$ *on* ***L*** *satisfies Axioms 1-3 and 6 if and only if there exists a a continuous function* $h(u,L(u))$ *where h is non-decreasing in L and* $h(u,0)=0$*, such that for all* $L_1, L_2 \in \boldsymbol{L}$,

(i) $$L_1 \succeq L_2 \Leftrightarrow \int_0^1 h(u,L_1(u))du \geq \int_0^1 h(u,L_2(u))du.$$

Proof. Assume that there exists a continuous function $h(u,L(u))$ which is non-decreasing in L and $h(u,0)=0$ such that (i) is true for all $\mathrm{L}_1, \mathrm{L}_2 \in \mathbf{L}$. Thus, from the expression

$$\int_0^1 \left(h(u,L_1(u)) - h(u,L_2(u)) \right) du$$

it follows by straightforward verification that $\succeq$ satisfies Axioms 1-3 and 6.

To prove sufficiency, note that ***L*** is analogous to a family of convex distribution functions. Furthermore, it follows from Axioms 1-3 and 6 that the conditions of Theorem 1 of Green and Jullien (1988) are satisfied and thus that there exists a continuous function $h(u,L(u))$ satisfying (i) where $h(u,0)=0$. It follows from the monotonicity property of Axiom 2 that $h(u,L(u))$ is non-decreasing in *L*. □

Now, let K be a functional, $K : \mathbf{L} \to [0,1]$ defined by

(3.4) $$K_h(L) = \int_0^1 h(u,L(u))du$$

It follows from Theorem 3.1 that K_h represents preferences that satisfy Axioms 1-3 and 6. The implication is that a social planner whose preferences satisfy Axioms 1-3 and 6 will choose among Lorenz curves so as to maximize K_h. For normalization purposes let h be such that $\int_0^1 h(u,u)du = 1$.

Accordingly, A_h defined by

(3.5) $$A_h = 1 - \int_0^1 h(u,L(u))du$$

measures the extent of inequality in an income distribution with Lorenz curve L when social preferences are consistent with Axioms 1-3 and 6, and takes the minimum value 0 iff incomes are equally distributed and the maximum value 1 iff one individual holds all income.

Further restriction on the preferences of the social planner can be introduced through the preference function h . By introducing the multiplicative specification $h(u,L(u)) = cp'(u)q(L(u))$ where q is a non-decreasing function in L, p' is the derivative of a positive monotonous function p defined on the unit interval and c is a normalization constant defined by $c = \left[\int q(u)dp(u)\right]^{-1}$, we get the following general family of rank-dependent measures of inequality $J_{p,q}$, defined by

(3.6) $$J_{p,q}(L) = 1 - c\int_0^1 q(L(u))dp(u).$$

The constant c and the normalization condition $q(0) = 0$ ensures that $J_{p,q}$ has the unit interval as its range, taking the maximum value 1 if one unit holds all income. Note that c is positive when p is non-decreasing and negative when p is non-increasing.

Since Axiom 6 represents a weakening of Axiom 4 as well as of Axiom 5, the family $J_{p,q}$ of inequality measures can be considered as a generalization of the families $\tilde{J}_p$ and J_q^* that allows the weights to depend on the magnitudes of income shares as well as on their rank in the distribution of income.

Note that even though $J_{p,q}(L)$ defined by (3.6) coincides with Quiggin's (1982) general family of rank-dependent criteria for choice under uncertainty when the Lorenz curve L is replaced by the distribution function F, the axiomatic theories of Quiggin (1982, 1989), Green and Jullien (1988) and Segal (1989) cannot be used to justify $J_{p,q}(L)$ as criteria for ranking Lorenz curves. However, as indicated in footnote 9 and demonstrated by Yaari (1987, 1988) and Aaberge (2001), the subfamily $\tilde{J}_p$ defined by (3.2) can either be justified as a theory for ranking income distributions or as a theory for ranking Lorenz curves.

As is generally acknowledged, measures of inequality are required to satisfy the *Pigou-Dalton principle of transfers*, which states that an income transfer from a richer to a poorer individual reduces income inequality, provided that their rank in the income distribution are unchanged. As is stated in Theorem 3.2 below, the Pigou-Dalton principle of transfers is equivalent to the condition of dominating non-intersecting Lorenz curves. A social planner who prefers the dominating one of non-intersecting Lorenz curves favours transfers of incomes which reduce the differences between the income shares of the donor and the recipient, and is therefore said to be *inequality averse*.

Definition 3.1. *A Lorenz curve L_1 is said to* ***first-degree dominate*** *a Lorenz curve L_2 if*

$$L_1(u) \geq L_2(u) \quad \textit{for all} \quad u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Theorem 3.2. (Fields and Fei (1978), Yaari (1988) and Aaberge (2001)). *Let L_1 and L_2 be Lorenz curves. Then the following statements are equivalent,*

(i) *L_1 first-degree dominates L_2*

(ii) *L_1 can be obtained from L_2 by a sequence of Pigou-Dalton progressive transfers*

(iii) $\tilde{J}_p(L_1) \leq \tilde{J}_p(L_2)$ *for all positive non-increasing p*

(iv) $J_q^*(L_1) \leq J_q^*(L_2)$ *for all positive non-decreasing q*

We refer to Fields and Fei (1978) for a proof of the equivalence between (i) and (ii),[12] Yaari (1988) for a proof of the equivalence between (i) and (iii), and Aaberge (2001) for a proof of the equivalence between (i) and (iv).

It follows from Theorem 3.2 that inequality aversion for $\tilde{J}_p$-measures and J_q^*-measures are characterized by positive non-increasing *p*-functions and positive non-decreasing *q*-functions. As demonstrated by Theorem 3.3, $J_{p,q}$ is consistent with inequality averse social preferences if and only if *cp* is positive non-decreasing and *q* is positive non-decreasing. Note that the equivalence between (i), (iii) and (iv) in Theorem 3.2 can be considered as special cases of the equivalence between (i) and (iii) in Theorem 3.3.

Theorem 3.3. *Let L_1 and L_2 be Lorenz curves. Then the following statements are equivalent,*

(i) *L_1 first-degree dominates L_2*

(ii) *L_1 can be obtained from L_2 by a sequence of Pigou-Dalton progressive transfers*

(iii) $J_{p,q}(L_1) \leq J_{p,q}(L_2)$ *for all positive non-decreasing cp and positive non-decreasing q.*

[12] See Rothschild and Stiglitz (1973) for a proof of the equivalence between (i) and (ii) in the case where the rank-preserving condition is abandoned in the definition of the Pigou-Dalton principle of transfers.

Proof. Since the equivalence of (i) and (ii) follows from Theorem 3.2 it remains to prove that (i) and (iii) are equivalent conditions. If condition (i) holds then

$$J_{p,q}(L_2) - J_{p,q}(L_1) = c\int_0^1 \left[q(L_1(u)) - q(L_2(u))\right]dp(u) \geq 0 \tag{3.7}$$

for all positive and non-decreasing q and all positive and non-decreasing cp.

To prove the converse statement we assume that (3.7) is satisfied for positive non-decreasing cp and positive non-decreasing q. By applying Lemma 1 given in the Appendix we then get that

$$q(L_1(u)) \geq q(L_2(u)) \text{ for all } u \in [0,1],$$

which implies that $L_1(u) \geq L_2(u)$ for all $u \in [0,1]$. □

By relying on (3.6) rather than on (3.2) or (3.3), we get measures of inequality that combine the features captured by $\tilde{J}_p$ and J_q^*. For example, by choosing $p_{1,k}(u) = 1 - u^k$, $q_{1,j}(t) = t^j$ and $c = -(j+k)/k$ in (3.6) we obtain the following subfamily of $J_{p,q}$,

$$J_{1,j,k}(L) = 1 + \frac{j+k}{k}\int_0^1 L^j(u)d(1-u^k) = 1 - (j+k)\int_0^1 u^{k-1}L^j(u)du, \quad j,k = 1,2,\ldots. \tag{3.8}$$

Note that the $J_{1,j,k}$-measures defined by (3.8) are more sensitive to changes that occur at the central and upper part than at the lower part of the income distribution (and the Lorenz curve). Moreover, the sensitivity of $J_{1,j,k}$ to changes that occur in the upper part of the income distribution increases with increasing *j* and/or *k*. By contrast, $J_{p,q}$-measures that places greater relative weight on changes that occur at the lower part of the income distribution are obtained by choosing the following specifications[13] $p_{2,k}(u) = (1-u)^k$, $q_{2,j}(t) = 1-(1-t)^j$ and $c = -(j+k)/j$ in (3.6), which yields

[13] Note that the choice $p(u) = (1-u)^k$ in (3.2) corresponds to the extended Gini family of inequality measures introduced by Donaldson and Weymark (1980).

$$J_{2,j,k}(L) = 1 + \frac{j+k}{j}\int_0^1 \left[1-(1-L(u))^j\right] d(1-u)^k =$$

(3.9)

$$\frac{(j+k)k}{j}\int_0^1 (1-u)^{k-1}(1-L(u))^j\,du - \frac{k}{j},\ \ j,k = 1,2,....$$

The sensitivity of $J_{2,j,k}$ to changes that occur in the lower part of the income distribution increases with increasing j and/or k. It follows from Theorem 3.3 that the $J_{1,j,k}$-measures as well as the $J_{2,j,k}$-measures satisfy the Pigou-Dalton principle of transfers.

Assume that $Z \sim F$. By inserting for

(3.10) $$\frac{E\left(Z \middle| Z \le F^{-1}(u)\right)}{EZ} = \frac{L(u)}{u}$$

and

(3.11) $$\frac{E\left(Z \middle| Z \ge F^{-1}(u)\right)}{EZ} = \frac{1-L(u)}{1-u}$$

in (3.8) and (3.9), respectively, we get the following alternative expressions for $J_{1,j,k}$ and $J_{2,j,k}$,

$$J_{1,j,k}(L) = \int_0^\infty \left\{1-\left[\frac{E\left(Z \middle| Z \le z\right)}{EZ}\right]^j\right\} dF^{j+k}(z) =$$

(3.12)

$$E\left\{1-\left[\frac{E\left(Z \middle| Z \le \max(Z_1, Z_2, ..., Z_{j+k})\right)}{EZ}\right]^j\right\},\quad j,k = 1,2,...,$$

and

$$J_{2,j,k}(L)=\frac{k}{j}\int_0^{\infty}\left\{\left(\frac{E\left(Z\middle|Z\ge z\right)}{EZ}\right)^j-1\right\}d\left(1-\left(1-F(z)\right)^{j+k}\right)=$$

(3.13)

$$\frac{k}{j}\left\{E\left[\frac{E\left(Z\middle|Z\ge\min\left(Z_1,Z_2,...,Z_{j+k}\right)\right)}{EZ}\right]^j-1\right\},\quad j,k=1,2,...,$$

where $Z_1,Z_2,...,Z_{j+k}$ is a random sample from *F*.

Note that the integrand of the first term of expression (3.12) is equal to the relative difference between the overall mean income raised by *j* and the average income raised by *j* of those units with income lower than the richest as we move up the distribution *F*. Thus, the $J_{1,j,k}$-measure is equal to the mean of these gaps. Alternatively, by relying on the second term of expression (3.12) we see that the $J_{1,j,k}$-measure can be interpreted as the average of relative gaps between the overall mean raised by *j* and the average income raised by *j* of those units with income lower than the maximum income of a random sample of size $j+k$ drawn from *F*. The average of these gaps is obtained by drawing a set of samples, each of size $j+k$ and compute the relative gaps for each of them. For a given *j* and *k* the $J_{1,j,k}$-measure is equal to the average of these relative gaps. Similarly, we see from expression (3.11) that the $J_{2,j,k}$-measure is determined by the relative difference between the average income raised by *j* of those income units with incomes higher than the minimum income of a sample of size $j+k$ drawn from *F* and the overall mean income raised by *j*. Thus, roughly spoken $J_{1,j,k}$ and $J_{2,j,k}$ show to what extent the overall mean is affected when respectively the highest and the lowest incomes are removed from the income distribution.

3.3. Mean normalized measures of inequality

Most measures of inequality are, as the Gini coefficient, normalized with respect to the mean, whereas the $J_{1,j,k}$-measures and the $J_{2,j,k}$-measures are normalized with respect to the mean raised by *j*. As will be demonstrated in Section 3.4, mean normalized versions of $J_{1,j,k}$ and $J_{2,j,k}$ can be used to define social welfare functions that proves useful for deriving measures of income mobility. A mean normalized version of $J_{1,j,k}$ is obtained from the following transformation

(3.14) $$I_{i,j,k}(L)=1-(1-J_{i,j,k}(L))^{\frac{1}{j}},$$

which yields

$$I_{1,j,k}(L) = 1 - \frac{\left\{ E\left[E\left(Z \middle| Z \le \max(Z_1, Z_2, ..., Z_{j+k}) \right) \right]^j \right\}^{\frac{1}{j}}}{EZ}, \quad j,k = 1,2,... \tag{3.15}$$

and a mean normalized version of $J_{2,j,k}$ is given by

$$I_{2,j,k}(L) = \left[\left(\frac{j+k}{k} \right)^{\frac{1}{j}} - 1 \right]^{-1} \left[\left(\frac{j}{k} J_{2,j,k}(L) + 1 \right)^{\frac{1}{j}} - 1 \right] =$$

$$\left[\left(\frac{j+k}{k} \right)^{\frac{1}{j}} - 1 \right]^{-1} \left(\frac{\left\{ E\left[E\left(Z \middle| Z \ge \min\left(Z_1, Z_2, ..., Z_{j+k} \right) \right) \right]^j \right\}^{\frac{1}{j}}}{EZ} - 1 \right), \quad j,k = 1,2,... \tag{3.16}$$

It follows from (3.14) that there is one to one correspondence between $I_{i,j,k}$ and $J_{i,j,k}$, which means that $I_{i,j,k}$ and $J_{i,j,k}$ produce identical rankings of Lorenz curves and that Theorem 3.2 also is valid for the mean normalized $I_{i,j,k}$-measures. It is also evident that by increasing *k*, $I_{1,j,k}$ becomes more sensitive to changes in the upper part of the income distribution, as fewer high incomes are removed from the income distribution. If we increase *j*, $I_{1,j,k}$ places even more weight on such changes, because the average income lower than the maximum income of the random sample of size *j*+*k* is raised by *j*. By the same token, the sensitivity of $I_{2,j,k}$ to changes in the lower part of the income distribution increases with increasing *j* and/or *k*.

Note that (3.15) and (3.16) for *j*=*k*=*1* provide two new alternative interpretations of the Gini coefficient. Moreover, the average of $I_{1,1,1}$ and $I_{2,1,1}$ provides the following third alternative interpretation of the Gini coefficient,

$$G = \frac{1}{2} \left\{ \frac{EE\left(Z \middle| Z \ge \min\left(Z_1, Z_2 \right) \right) - EE\left(Z \middle| Z \le \max\left(Z_1, Z_2 \right) \right)}{EZ} \right\}. \tag{3.17}$$

Table 1 provides estimates of the Gini-coefficient, according to (3.15), (3.16) and (3.17), respectively. The data is generated from a uniform distribution with incomes ranging from 0 to 1000. This simulation example provides further intuition behind the proposed family of rank-dependent inequality measures in general, and the new expressions for the Gini-coefficient in particular. By repeatedly computing how the average income is affected when removing incomes higher than the highest income or lower than the lowest income from random draws of two incomes, we obtain three alternative and intuitive appealing interpretations of the Gini-coefficient. In a similar vain, the other members of the proposed rank-dependent family of inequality measures reflect to what extent the average income is affected by removing the highest or lowest incomes from the income distribution.

Table 1. Alternative estimates of the Gini-coefficient

Income draw: (Z_1, Z_2)	$E[Z \mid Z \leq \max(Z_1,Z_2)]/EZ$	$E[Z \mid Z \geq \min(Z_1,Z_2)]/EZ$	G From (3.15)	G from (3.16)	G from (3.17)
(914, 760)	.914	1.764	.086	.764	.425
(620, 354)	.626	1.352	.374	.352	.363
(899, 636)	.901	1.642	.099	.642	.370
(423, 78)	.425	1.077	.575	.077	.326
(848, 497)	.849	1.498	.151	.498	.324
(346, 129)	.345	1.129	.655	.129	.392
(316, 243)	.317	1.242	.683	.242	.463
(287, 252)	.290	1.253	.710	.253	.481
(254, 96)	.257	1.095	.743	.095	.418
(955, 713)	.956	1.717	.044	.717	.381
Estimate of the Gini-coefficient (10 income draws)			.412	.377	.394
Estimate of the Gini-coefficient (100 income draws)			.330 (.026)	.358 (.025)	.344 (.011)

Note: The income data is generated from random draws of 1000 observations from a uniform distribution defined over the interval [0, 1000]. Column 1 shows the realizations from random draws of two incomes. Column 2 computes the average income of the observations with income lower than the maximum of this pair of incomes. Column 3 computes the average income of the observations with income higher than the minimum of this pair of incomes. The first ten rows of column 4-6 compute the Gini-coefficient by each pair of income draws, according to (3.15), (3.16) and (3.17), respectively. The last two rows provide estimates of the Gini-coefficient for 10 and 100 pair of income draws, according to (3.15), (3.16) and (3.17), respectively. Standard errors in parentheses are computed as the sample standard deviation divided by the square root of the number of the number of income draws.

As is evident from (3.17), the Gini coefficient can be considered as a "tail-symmetric" measure of inequality. Moreover, expression (3.16) might be used to justify the standard claim (see Atkinson, 1970) that the Gini coefficient is more sensitive to changes that take place in the central part of the distribution than at the tails. It should be noted, however, that this property is, as demonstrated by Aaberge (2000), true only for unimodal distributions that are neither strongly skew to the left or to the right, provided that transfers sensitivity is defined according to Kolm's (1976) principle of diminishing transfers. Alternative "tail-symmetric" measures of inequality to the Gini coefficient are given by the average of $I_{1,k,k}$ and $I_{2,k,k}$ defined by

(3.18) $$S_k = \frac{1}{2}\left\{\frac{\left\{E\left[E\left(Z \middle| Z \geq \min(Z_1, Z_2, ..., Z_{2k})\right)\right]^k\right\}^{\frac{1}{k}} - \left\{E\left[E\left(Z \middle| Z \leq \max(Z_1, Z_2, ..., Z_{2k})\right)\right]^k\right\}^{\frac{1}{k}}}{EZ}\right\},$$

k=1, 2,…

Thus, $\{S_k : k = 1,2,...\}$ can be considered as a tails-sensitive family of generalized Gini inequality measures where the tails-concern increases with increasing k. Roughly spoken, the S_k inequality measures display the relative difference in k-order conditional means when respectively the lowest and the highest incomes are removed from the population.

3.4. Inequality and social welfare

Theoretically based measures of social welfare that admit a decomposition with respect to average income (or average individual welfare) and inequality are considered particular attractive since they offer an explicit treatment of the trade-off between "the size and the distribution of the cake". As demonstrated by Yaari (1988), members of the family of rank-dependent inequality measures (3.3) are associated with the following social welfare functions

(3.19) $$\tilde{W}_p = \int_0^1 p(u)F^{-1}(u)du = \mu(1 - \tilde{J}_p(L)),$$

where μ and L are the mean and the Lorenz curve of F.

Note that the normative justification of (3.19) can be made in terms of a theory for ranking the distribution functions (F), as proposed by Yaari (1987), or by considering the last term of (3.19) as a value judgement of the trade-off between the mean and (in)equality in deriving social welfare functions, as proposed by Ebert (1987). A mean-independent ordering of income distributions in terms of inequality, i.e. an ordering of Lorenz curves, forms the basis of Ebert's approach.[14] Since $I_{1,j,k}$ (equivalent to $J_{1,j,k}$) and $I_{2,j,k}$ (equivalent to $J_{2,j,k}$) can be considered as alternative representations of Lorenz curve orderings, we can use Ebert's approach as a basis for introducing the following social welfare functions

[14] See Aaberge (2001) for a theory of ranking Lorenz curves.

(3.20) $$W_{1,j,k} = EZ\left(1 - I_{1,j,k}(L)\right) = \left\{ E\left[E\left(Z \middle| Z \le \max\left(Z_1, Z_2, ..., Z_{j+k}\right)\right)\right]^j \right\}^{\frac{1}{j}}, \quad j,k = 1,2,...,$$

and

(3.21)

$$W_{2,j,k} = EZ\left(1 - I_{2,j,k}(L)\right) =$$

$$EZ - \left(\left[\left(\frac{j+k}{k}\right)^{\frac{1}{j}} - 1\right]^{-1}\right)\left(\left\{ E\left[E\left(Z \middle| Z \ge \min\left(Z_1, Z_2, ..., Z_{j+k}\right)\right)\right]^j \right\}^{\frac{1}{j}} - EZ\right), \; j,k = 1,2,...$$

Note that $W_{i,j,k} \le EZ$ and that $W_{i,j,k} = EZ$ if and only if the incomes are equally distributed. Thus, $W_{i,j,k}$ can be interpreted as *the equally distributed equivalent income*, and the product $(EZ)I_{i,j,k}(L)$ as a measure of the loss in social welfare due to inequality in the distribution of permanent income.

4. Rank-dependent measures of income mobility based on permanent incomes

This section introduces a family of measures of income mobility that rely on *(i)* the permanent income measure introduced in Section 2, and *(ii)* the generalized family of rank-dependent measures of income inequality introduced in Section 3. In the general case, we allow for individual-specific interest rates on borrowing and saving as well as for liquidity constraints in determining the permanent income. Thus, our measure of permanent income incorporates the cost of making inter-period income transfers, and hence account for the welfare loss that may be associated with income fluctuations. Consequently, high mobility will be, everything else equal, strictly socially preferable. The encompassing nature of the proposed family of generalized rank-dependent measures of income mobility is directly linked to the alternative specifications of the credit marked and the intertemporal preference structure.

4.1. A generalized family of rank-dependent measures of income mobility

Let L_Z and L_{Z_R} be the Lorenz curves for the distribution of the observed permanent income Z and the distribution of the hypothetical reference permanent income Z_R when there is no mobility. The latter distribution is formed by assigning the lowest income in every period to the poorest individual in the first period, the second lowest to the second poorest, and so on. Accordingly, the design of the distribution of Z_R does not alter the marginal period-specific distributions. Since L_Z can be attained from L_{Z_R} by a sequence of period-specific Pigou-Dalton income transfers we have that $L_Z(u) \geq L_{Z_R}(u)$ for all $u \in [0,1]$, and moreover that $L_Z(u) = L_{Z_R}(u)$ for all u if and only if Z is equal to Z_R. Accordingly, $L_Z(u) - L_{Z_R}(u)$ can be used to analyse income mobility. However, in order to quantify the degree of mobility underlying a distribution of permanent incomes defined over a given period it is necessary to introduce summary measures of mobility. Since $J_{p,q}$ defined by (3.6) for all positive and non-decreasing *cp* and all positive and non-decreasing *q* preserves first-degree Lorenz dominance, it appears attractive to use this family of generalized rank-dependent measures of inequality as a basis for defining the following family of rank-dependent measures of mobility

(4.1)
$$M_{p,q}(L_Z) = \frac{J_{p,q}(L_{Z_R}) - J_{p,q}(L_Z)}{J_{p,q}(L_{Z_R})}.$$

It is straightforward to verify that $0 \leq M \leq 1$, with strict equality if and only if the distribution of permanent income Z is equal to the distribution of the reference permanent income Z_R. Thus, the state of no mobility is defined to occur when the individuals' positions in the short-term income distributions are constant over time. Mobility is measured as relative reduction of the inequality in the distribution of permanent income for a given period due to changes in the individuals' positions and incomes shares in the short-term distributions of income. By explicitly incorporating the cost of making inter-period income transfers in M, and thus the welfare loss that may be associated with income fluctuations, high mobility will be everything else equal strictly socially preferable. Hence, we accommodate the most common criticism measures of mobility as an equalizer of long-term income, namely that high mobility may imply income instability for the individual which will matter for his or her welfare if it is costly to transfer income. Note also that the measure of income mobility defined by (4.1) allows for individual-specific interest rates on saving and borrowing as well as for liquidity constraints.

4.2. Measuring income mobility based on average income

The method for measuring mobility defined by (4.1) can be considered as a generalization of the standard measure of income mobility as an equalizer of long-term income, where the average real income over a sequence of periods is used as a measure of permanent incomes. By assuming that the rates of time preferences and the real interest rates are equal to zero in each period, i.e.

$r_2 = r_3 = \cdots = r_T = \delta = 0$, we get that the equally allocated equivalent income Z is equal to the average income $\bar{Y}$,

(4.2)
$$Z = \bar{Y} = \frac{1}{T}\sum_{t=1}^{T} Y_t .$$

Thus, using the average income as a measure of the equally allocated equivalent income means that possible costs and benefits of receiving income at different times are disregarded.

Let $\mu_t = EY_t$ and $\mu = \sum_{t=1}^{T} \mu_t$. Since $L_{\bar{Y}}(u) = \sum_{t=1}^{T} \frac{\mu_t}{\mu} L_{Y_t}(u)$ when there is no mobility, the Mehran-Yaari subfamily of rank-dependent measures of inequality admits the following decomposition

(4.3)
$$\tilde{J}_p(L_Z) = \sum_{t=1}^{T} \frac{\mu_t}{\mu} \tilde{J}_p(L_{Y_t})$$

when the average income forms the basis for measuring inequality. In this case, we get the following special case for (4.1)

(4.4)
$$\tilde{M}_p(L_{\bar{Y}}) = \frac{\sum_{t=1}^{T} \frac{\mu_t}{\mu} \tilde{J}_p(L_{Y_t}) - \tilde{J}_p(L_{\bar{Y}})}{\sum_{t=1}^{T} \frac{\mu_t}{\mu} \tilde{J}_p(L_{Y_t})} .$$

Mobility measures based on (4.4) may be interpreted as the relative reduction in inequality over the extended accounting period of income due to changes in the period rankings (and incomes) of the individuals over time, when it is assumed to be costless to make income-transfers across periods. Consequently, analyses based on (4.4) run the risk of mixing the equalising effect of high mobility with the loss of welfare from fluctuating income. Thus, high mobility, everything else equal, is no longer necessarily desirable from the perspective of the social planner.

4.3. Measuring income mobility based on annuity income

Making incomes from different periods comparable is not merely a question of accounting for changes in the price of goods; it is also necessary to take the price of consumption into account. The price of consumption depends on the real interest rates, which determine how much consumption an individual must give up in the future for being able to consume more today. Thus, it appears more appropriate to use the annuity value A defined by (2.8) rather than the average income as a basis for measuring income mobility. Let A_R denote A in the case where the observed distribution of income streams are replaced by the hypothetical reference distribution..

Replacing Z with A and Z_R with A_R in (4.1) yields

(4.5)
$$M_{p,q}(L_A) = \frac{J_{p,q}(L_{A_R}) - J_{p,q}(L_A)}{J_{p,q}(L_{A_R})}.$$

Similarly as for the average income case, the following convenient expression emerges for the Mehran-Yaari family of mobility measures

(4.6)
$$\tilde{M}_p(L_A) = \frac{\sum_{t=1}^{T} \frac{\mu_t}{\mu} b_t \tilde{J}_p(L_{Y_t}) - \tilde{J}_p(L_A)}{\sum_{t=1}^{T} \frac{\mu_t}{\mu} b_t \tilde{J}_p(L_{Y_t})},$$

where

(4.7)
$$b_t = \frac{\prod_{j=t+1}^{T}(1+r_j)}{1+\sum_{s=1}^{T-1}\prod_{j=s+1}^{T}(1+r_j)}, \quad t = 1,2,\ldots,T-1, \; b_T = 1.$$

At first sight, it may seem like mobility analyses based on (4.5) and (4.6) relies on the controversial assumption that the social planner sets the rate of time preferences equal to the real interest rates. However, as follows from Theorem 4.1 the mobility measure $M_{p,q}(L_Z)$ proves to be *independent* of the social planner's choice of preference parameter values provided that the functional form of the instantaneous utility function in (2.5) is of the Bergson type.

Theorem 4.1. Let the permanent income Z be defined by (2.6) where the maximum utility level U is given by (2.5) and the instantaneous utility function u is defined by

(i)
$$u(x) = \begin{cases} \frac{1}{1-\varepsilon}(x^{1-\varepsilon} - 1) & \text{if } \varepsilon \neq 1 \\ \log x & \text{if } \varepsilon = 1. \end{cases}$$

Then

(ii)
$$M_{p,q}(L_Z) = M_{p,q}(L_A).$$

Proof. It follows from Theorems 2.1 and 2.2 that *Z* under the specification (i) in Theorem 4.1 for *u* is given by $Z = k(\varepsilon,\delta)A$. Since $J_{p,q}$ is invariant with respect to scale transformations of the random variable in question we have that $J_{p,q}(L_Z) = J_{p,q}(L_A)$ and $J_{p,q}(L_{Z_R}) = J_{p,q}(L_{A_R})$. □

Theorem 4.1 shows that $M_{p,q}(L_Z)$ will be equal to $M_{p,q}(L_A)$ when the instantaneous utility function is of the Bergson type. This implies that the annuity income may form the basis for measuring mobility, even if the rate of time preference differ from the real interest rates, and thus the preferred consumption levels vary over time.

4.4. A social welfare approach for measuring mobility

Since $\mu_Z = \mu_{Z_R}$ when $Z = A$, the annuity based mobility measure defined by (4.5) can be given the following alternative expressions in terms of the social welfare functions defined by (3.20) and (3.21), provided that the measurement of inequality is based on respectively (3.15) and (3.16) rather than on (3.12) and (3.13),

$$(4.8) \qquad M_{i,j,k}(L_A) = \frac{I_{i,j,k}(L_{Z_R}) - I_{i,j,k}(L)}{I_{i,j,k}(L_{Z_R})} = \frac{W_{i,j,k} - W_{i,j,k}^R}{EZ - W_{i,j,k}^R}, \quad i = 1,2 \; and \; j,k = 1,2,\dots,$$

where $W_{i,j,k}^R$ is the social welfare attained by the welfare function $W_{i,j,k}$ when there is no income mobility. The numerator of (4.8) provides a measure of the gain in social welfare due to income mobility, whereas the denominator gives a measure of maximum attainable gain in social welfare due to income mobility when $W_{i,j,k}$ is used as a measure of social welfare. Thus, the social welfare produced by the observed distribution *F* of permanent income *Z* admits the following decomposition,

$$(4.9) \qquad W_{i,j,k} = W_{i,j,k}^R + M_{i,j,k}(EZ - W_{i,j,k}^R),$$

where the first term gives the level of social welfare attained when there is no mobility and the second term expresses the contribution to social welfare due to income mobility.

By inserting the second terms of (3.20) and (3.21) in (4.8) we get the following expressions for $M_{1,j,k}$ and $M_{2,j,k}$

(4.10)

$$M_{1,j,k} = \frac{\left\{E\left[E\left(Z\middle|Z \le \max\left(Z_1, Z_2, ..., Z_{j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}} - \left\{E\left[E\left(Z_R\middle|Z_R \le \max\left(Z_{R,1}, Z_{R,2}, ..., Z_{R,j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}}}{EZ - \left\{E\left[E\left(Z_R\middle|Z_R \le \max\left(Z_{R,1}, Z_{R,2}, ..., Z_{R,j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}}}$$

and

$$(4.11)\quad M_{2,j,k} = \frac{\left\{E\left[E\left(Z_R\middle|Z_R \ge \min\left(Z_{R,1}, Z_{R,2}, ..., Z_{R,j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}} - \left\{E\left[E\left(Z\middle|Z \ge \min\left(Z_1, Z_2, ..., Z_{j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}}}{\left\{E\left[E\left(Z_R\middle|Z_R \ge \min\left(Z_{R,1}, Z_{R,2}, ..., Z_{R,j+k}\right)\right)\right]^j\right\}^{\frac{1}{j}} - EZ}$$

5. Summary and discussion

The notion of mobility considered in this paper has its origin from Friedman's (1962) discussion of the relationship between income mobility and long-term income inequality. This relationship has motivated a considerable theoretical and applied literature, starting with Shorrocks (1978), who employed a two-step aggregation approach to assess mobility as an equalizer of long-term income. The first step consists of aggregating the income stream of each individual into an interpersonal comparable measure of permanent income, whereas the second step deals with the problem of aggregating the distribution of permanent incomes into measures of social welfare, inequality, and mobility.

In Shorrocks (1978) as well as in most subsequent empirical studies of income mobility, the average real income over several years is used as an approximation for permanent income. A common objection against this approach, is the fact that it ignores that high mobility may imply income instability for the individuals, which will matter for their welfare if it is costly to transfer income. By disregarding the welfare loss that may be associated with income fluctuations, it is not necessarily true that high mobility will be preferable for an inequality averse social planner. To address this issue, we introduce a measure of permanent income that explicitly incorporates the costs of and constraints on making inter-period income transfers. [15] If the instantaneous utility function of the intertemporal utility function belongs to the much used Bergson family, our permanent income measure is equivalent to the concept of utility-equivalent annuity used by Nordhaus (1973) and Creedy (1999) as a measure of permanent income. Nordhaus (1973) as well as Creedy (1999) express concerns about the sensitivity of analysis of lifetime inequality to the social planner's choice of preference parameters. In this paper, we demonstrate, however, that their concern is uncalled for, as inequality and mobility estimates based on utility-equivalent annuity measures prove to be independent of these preference parameters.

After aggregating the incomes of an individual into a permanent income measure, we introduce a method for aggregating the permanent incomes across individuals into measures of long-term income inequality, social welfare and income mobility, when immobility is defined as no changes over time in individuals' rank in the short term distributions of income. Since this definition calls for measures of mobility that are derived from rank-dependent measures of inequality, we employ an axiomatic approach to justify the introduction of a generalized family of rank-dependent measures of inequality where the distributional weights, as opposed to the members of the family introduced by Mehran (1976) and Yaari (1988), depend on income shares as well as on population shares. Importantly for empirical research, it is straightforward to estimate these inequality measures, which supplement each other with regard to sensitivity to changes in the lower, the central and the upper part of the income distribution. A subfamily of this generalized family of rank-dependent measures of inequality is shown to be associated with social welfare functions that prove to have intuitively appealing interpretations. Further, the general family of inequality measures provides several new interpretations of the Gini coefficient.

While our paper has considered mobility as the extent to which equalisation of income occurs as the accounting period is extended, Chakravarty et al. (1985), Ruiz-Castello (2004) and Fields (2009) proposes mobility measures that tell us how inequality of permanent incomes compares with the inequality of the first-year incomes. In the latter case, the mobility measures are viewed as a welfare comparison between the actual path of the income distribution and a hypothetical path where there is no change over time from the first-year distribution. A possible advantage of these mobility measures is that they convey whether mobility equalises or disequalises the distribution of permanent income relative to the first-year distribution (see Benabou and Ok, 2001). It is, however, straightforward to apply our measure of permanent income as well as the proposed family of rank-dependent measures to construct measures of mobility defined as equalisers of long-term incomes relative to first-year

[15] An alternative approach to capture the effect of income fluctuations is found in Maasoumi and Zandvakili (1986, 1990), who replace individual's average income by a measure of the utility of his incomes, where the incomes of the different periods are treated as distinct and substitutable attributes depending on the choice of elasticity of substitution. The role that credit markets play in this measure of permanent income is however not clear, as it is not derived from intertemporal choice theory but the upshot of an aggregator function that is derived by appealing to a generalised criterion from information theory.

incomes. This is simply achieved by replacing of the inequality in the distribution of permanent income derived under the assumption of no changes in individuals' positions over time with the inequality in the distribution of the initial year income in the denominator of the proposed family of rank-dependent measures of mobility. By doing so, we can separate equalising income mobility from income instability costly for individuals, using first-year income distribution as the base.

6. References

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7. Appendix

LEMMA 1. *Let H be the family of bounded, continuous and non-negative functions on [0,1] which are positive on* $\langle 0,1\rangle$ *and let g be an arbitrary bounded and continuous function on [0,1]. Then*

$$\int g(t)\, h(t)\, dt > 0 \quad \text{for all} \quad h \in H$$

implies

$$g(t) \geq 0 \quad \text{for all} \quad t \in [0,1]$$

and the inequality holds strictly for at least one $t \in \langle 0,1\rangle$.

The proof of Lemma 1 is known from mathematical textbooks.