Social Foundation of Nash Bargaining Solution

In-Koo Cho[1] Akihiko Matsui[2]

[1]Department of Economics
University of Illinois

[2]Faculty of Economics
University of Tokyo

May 24, 2010

Society

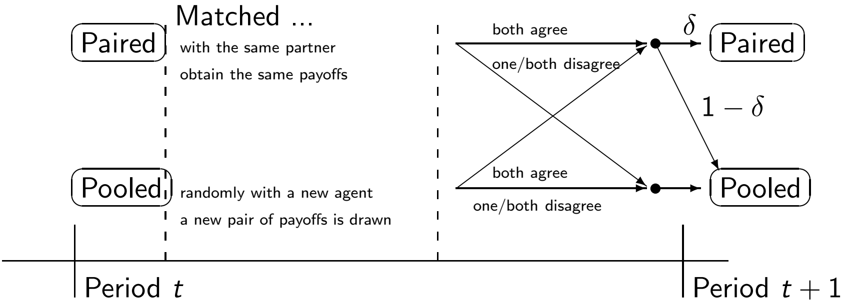
Society

- Imagine a society populated with a continuum of rational players.

Society

- Imagine a society populated with a continuum of rational players.
- The players have specific roles, say row and column players. They are matched, form a partnership and separates the relationship to look for another relationship. Let $u_i(s_i, s_j)$ be the intrinsic payoff function of the underlying bargaining problem $\langle S, v^0 \rangle$.

Timing of Matches and Decisions



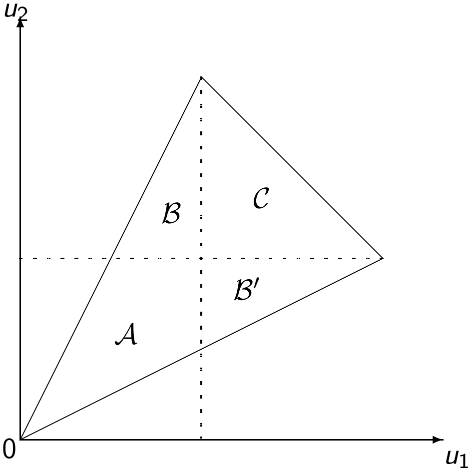


Figure: Battle of the Sexes

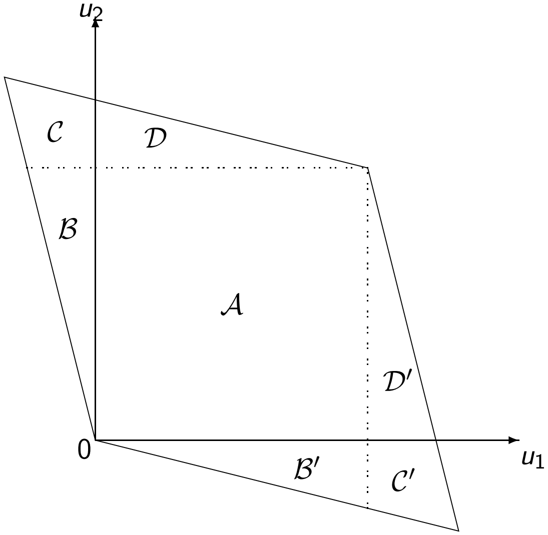


Figure: Prisoner's dilemma

Observation

Observation

- Suppose that an outsider (i.e., an economist) observes that the behavior of the agents lead to Nash bargaining solution. That is, whenever two player reaches an agreement, they appear to split the surplus according to the Nash bargaining solution.

Observation

- Suppose that an outsider (i.e., an economist) observes that the behavior of the agents lead to Nash bargaining solution. That is, whenever two player reaches an agreement, they appear to split the surplus according to the Nash bargaining solution.
- How can we explain such an outcome as a result of optimization of the individual agents?

Social Preference

Social Preference

- Suppose that the individual player's optimization is based upon a social preference

$$\mathbf{u}_i(s_i, s_j) = \min(u_i(s_i, s_j) - v_i^0, u_j(s_i, s_j) - v_j^0)$$

instead of his intrinsic payoff function $u_i(s_i, s_j)$ where v_i^0 is the disagreement payoff of player i.

Social Preference

- Suppose that the individual player's optimization is based upon a social preference

$$\mathbf{u}_i(s_i, s_j) = \min(u_i(s_i, s_j) - v_i^0, u_j(s_i, s_j) - v_j^0)$$

  instead of his intrinsic payoff function $u_i(s_i, s_j)$ where v_i^0 is the disagreement payoff of player i.
- We can explain the Nash bargaining solution as an outcome of the social preference. Refining the solution concept, we can nail down the Nash bargaining solution as the unique solution of the game.

Social Preference

- Suppose that the individual player's optimization is based upon a social preference

$$\mathbf{u}_i(s_i, s_j) = \min(u_i(s_i, s_j) - v_i^0, u_j(s_i, s_j) - v_j^0)$$

instead of his intrinsic payoff function $u_i(s_i, s_j)$ where v_i^0 is the disagreement payoff of player i.

- We can explain the Nash bargaining solution as an outcome of the social preference. Refining the solution concept, we can nail down the Nash bargaining solution as the unique solution of the game.
- Using this prediction, we can carry out experiments to understand how the behavior of the individual agents leads to social outcomes and so on.

Pros and Cons

Pros and Cons

- This approach provides a flexible, yet parsimonious, model to test the behavior of economic agents. Flexible specification of social preference is extremely useful for econometric treatment of experimental data.

Pros and Cons

- This approach provides a flexible, yet parsimonious, model to test the behavior of economic agents. Flexible specification of social preference is extremely useful for econometric treatment of experimental data.
- Flexibility is often viewed as lack of discipline in constructing a model. To explain the discrepancy from the selfish behavior, some departure from a fully rational selfish player might be necessary, but theorists asked for a small departure to explain a large discrepancy.

This paper

This paper

- Social preference is represented by social outcome.

This paper

- Social preference is represented by social outcome.
- Social outcome is a steady state of social dynamics.

This paper

- Social preference is represented by social outcome.
- Social outcome is a steady state of social dynamics.
- Social dynamics is generated by repeated interactions among agents through long term relationship.

This paper

- Social preference is represented by social outcome.
- Social outcome is a steady state of social dynamics.
- Social dynamics is generated by repeated interactions among agents through long term relationship.
- Is it possible to support the Nash bargaining solution (a social preference) as a steady state of social dynamics, in which the agents are rational?

This paper

- Social preference is represented by social outcome.
- Social outcome is a steady state of social dynamics.
- Social dynamics is generated by repeated interactions among agents through long term relationship.
- Is it possible to support the Nash bargaining solution (a social preference) as a steady state of social dynamics, in which the agents are rational?
- We use the Nash bargaining solution as a laboratory to develop analytic tools to answer general questions.

This paper

- Social preference is represented by social outcome.
- Social outcome is a steady state of social dynamics.
- Social dynamics is generated by repeated interactions among agents through long term relationship.
- Is it possible to support the Nash bargaining solution (a social preference) as a steady state of social dynamics, in which the agents are rational?
- We use the Nash bargaining solution as a laboratory to develop analytic tools to answer general questions.
- By understanding the micro structure to sustain a social preference, we can tell which criterion is more plausible than others.

Difference

Difference

- In contrast to behavioral models, the decision of each player is driven by intrinsic payoff function.

Difference

- In contrast to behavioral models, the decision of each player is driven by intrinsic payoff function.
- In contrast to social learning models, players are rational and forward looking. The expectations of the future influence the present choice of individual agent in an important way.

Difference

- In contrast to behavioral models, the decision of each player is driven by intrinsic payoff function.
- In contrast to social learning models, players are rational and forward looking. The expectations of the future influence the present choice of individual agent in an important way.
- In contrast to repeated game models, we generate a sharp prediction to support the maxmin outcome over a large class of social interactions.

Results

Results

- As we are interested in a steady state of an economy, we focus on the stationary equilibrium, which does not use weakly dominated strategies.

Results

- As we are interested in a steady state of an economy, we focus on the stationary equilibrium, which does not use weakly dominated strategies.
- As the discount factor converges to 1, and as the exogenous probability of break down vanishes, any stationary undominated equilibrium in an economy populated with a continuum of agents sustain the Nash bargaining solution.

Results

- As we are interested in a steady state of an economy, we focus on the stationary equilibrium, which does not use weakly dominated strategies.
- As the discount factor converges to 1, and as the exogenous probability of break down vanishes, any stationary undominated equilibrium in an economy populated with a continuum of agents sustain the Nash bargaining solution.
- Much, if not all, of this result is carried over if the number of players is finite, and if we make other small changes to the game.

Description

Society consists of 2 units mass of players.

$$I = [0, 2).$$

$i \in [0, 1)$ is called a row player, and $j \in [1, 2)$ is called a column player.

At each period $t \geq 1$, some people are in the long term relationship with someone else:

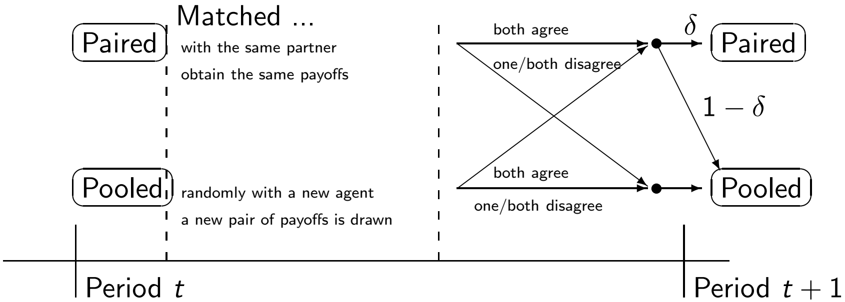
$$0 \leq i_1, \ldots, i_k \leq 1; \quad 1 \leq n + 1 \leq j_1, \ldots, j_k \leq 2,$$

$$\mathcal{P}_t = \{(i_1, j_1), \ldots, (i_k, j_k)\}$$

is the set of pairs in period t; and U_t^r (U_t^c) is the set of single row (column) players:

$$U_t = U_t^r \cup U_t^c.$$

Timing of Matches and Decisions



If $i \in U_t^r$,...

If $i \in U_t^r$,...

$i \in U_t^r$ is searching for a partner in U_t^c and vice versa.

If $i \in U_t^r$,...

$i \in U_t^r$ is searching for a partner in U_t^c and vice versa.

We assume that each player has an equal chance to be matched to another agent in a different population. If i meets j, both parties investigate whether they can initiate a long term relationship.

If $i \in U_t^r$,...

$i \in U_t^r$ is searching for a partner in U_t^c and vice versa.

We assume that each player has an equal chance to be matched to another agent in a different population. If i meets j, both parties investigate whether they can initiate a long term relationship.

Let

$$\langle S, v^0 \rangle$$

be the bargaining problem where S is compact and convex subset of $\mathbb{R}^2$ and $v^0 = (v_r^0, v_c^0)$ be the disagreement payoff vector.

If $i \in U_t^r$,...

$i \in U_t^r$ is searching for a partner in U_t^c and vice versa.

We assume that each player has an equal chance to be matched to another agent in a different population. If i meets j, both parties investigate whether they can initiate a long term relationship.

Let

$$\langle S, v^0 \rangle$$

be the bargaining problem where S is compact and convex subset of $\mathbb{R}^2$ and $v^0 = (v_r^0, v_c^0)$ be the disagreement payoff vector.

We often interpret v^0 as a one shot Nash equilibrium payoff vector.

Conditioned on (v_r, v_c),...

Conditioned on (v_r, v_c),...

1. $(v_r, v_c) \in V$ is randomly drawn according to a probability distribution with a full support over V.

Conditioned on (v_r, v_c),...

1. $(v_r, v_c) \in V$ is randomly drawn according to a probability distribution with a full support over V.
2. After consuming the util, each player decides whether or not to continue (A) or not continue (R) the partnership.
3. The partnership breaks down if at least one player chooses R, and $r \in U^r_{t+1}$ $c \in U^c_{t+1}$.
4. If both players chooses A, then with probability $1 - \delta$, an exogenous shock arrives and $r \in U^r_{t+1}$ $c \in U^c_{t+1}$.
5. If both players chooses A, then with probability δ, $\{r, c\} \in \mathcal{P}_{t+1}$.

Conditioned on (v_r, v_c),...

1. $(v_r, v_c) \in V$ is randomly drawn according to a probability distribution with a full support over V.
2. After consuming the util, each player decides whether or not to continue (A) or not continue (R) the partnership.
3. The partnership breaks down if at least one player chooses R, and $r \in U^r_{t+1}$ $c \in U^c_{t+1}$.
4. If both players chooses A, then with probability $1 - \delta$, an exogenous shock arrives and $r \in U^r_{t+1}$ $c \in U^c_{t+1}$.
5. If both players chooses A, then with probability δ, $\{r, c\} \in \mathcal{P}_{t+1}$.
6. If they fail to form a partnership, or the partnership breaks down, then $(u_{r,t}, u_{c,t}) = (v^0_r, v^0_c)$, $i \in U^r_{t+1}$, and $j \in U^c_{t+1}$.

If $(i, j) \in \mathcal{P}_t$,...

In period t, $(u_{r,t}, u_{c,t}) = (v_r, v_c)$. The only difference is that instead of any payoff vector from V, only $v = (v_r, v_c)$ is on the table. Each player has an option to dissolve R or continue A.

If at least one player chooses R, then both players are dumped to U.

If both players choose A, then with probability $\delta < 1$, $\{i, j\} \in \mathcal{P}_{t+1}$, and with probability $1 - \delta$, $i \in U^r_{t+1}$ and $j \in U^c_{t+1}$.

We make no presumption about how long the existing partnership lasts. The duration of the partnership is endogenous, and is the main focus of the analysis.

Search

We regard the probability distribution ν over V as a reduced form of a search and bargaining process, which is not specifically modeled. We impose a minimal restriction on ν, in order to cover a broad class of search process. As a result, we know little about the properties of the symmetric stationary equilibrium other than the efficiency.

Assumptions on ν

Probability distribution ν has a density function f_ν which is Lipschitz continuous and positive over V: $\exists H, L$ such that $\forall v, v' \in V$,

$$\left| \frac{f_\nu(v) - f_\nu(v')}{\|v - v'\|} \right| \leq H$$

and

$$f_\nu(v) \geq L > 0.$$

Assumptions on ν

Probability distribution ν has a density function f_ν which is Lipschitz continuous and positive over V: $\exists H, L$ such that $\forall v, v' \in V$,

$$\left| \frac{f_\nu(v) - f_\nu(v')}{\|v - v'\|} \right| \leq H$$

and

$$f_\nu(v) \geq L > 0.$$

Later, we admit a more general class of f_ν, which is almost necessary for the main result.

History and strategy

To simplify the model, we assume that when the two player are met, the private history of each player is not revealed to the other player. Let

$$h_{i,t} = (s_{i,1}, \ldots, s_{i,t-1})$$

be a private history of player i at t, where

$$s_{i,t} = (u_{i,t}, r_{i,t}).$$

$u_{i,t}$ is the payoff of agent i, $r_{i,t} \in \{A, R\}$ is the action, either "agree" (A) or "not agree" (R), taken by agent i, and q_t is the coalitional structure.
A (social) history at time t is given by

$$h_t = (s_1, \ldots, s_{t-1}).$$

Let H_i be the set of all private histories of player i. A strategy of agent $i \in I$ is a function

$$f_i : H_i \times \mathbb{R}^2 \to \{A, R\},$$

measurable with respect to i's information.
The payoff function of player i is given by

$$\mathrm{U}_i(f) = \mathrm{E}^f \left[(1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} u_{i,t} \right] \tag{2.1}$$

where E^f is the expectation operator induced by f, and $\beta \in (0,1)$ is a discount factor. The solution concept is sequential equilibrium.

Stationary

Communication among player does not exist. Thus, no player can observe what others do, or the state of the society. He only knows his private history.

Stationary

Communication among player does not exist. Thus, no player can observe what others do, or the state of the society. He only knows his private history.

As a first step of computing a (sequential) equilibrium, let us focus on a symmetric stationary equilibrium in which each player's action is conditioned on his state instead of entire history.

$$\Sigma_i = \{\emptyset\} \cup \{(j, v_i, v_j) | j \neq i, (v_i, v_j) \in V\}$$

Let W_i^0 and $W_i(v_i)$ be the value functions associated with the states.

Stationary

Communication among player does not exist. Thus, no player can observe what others do, or the state of the society. He only knows his private history.

As a first step of computing a (sequential) equilibrium, let us focus on a symmetric stationary equilibrium in which each player's action is conditioned on his state instead of entire history.

$$\Sigma_i = \{\emptyset\} \cup \{(j, v_i, v_j) | j \neq i, (v_i, v_j) \in V\}$$

Let W_i^0 and $W_i(v_i)$ be the value functions associated with the states.

Guess an equilibrium outcome where $(r^*, \ldots, r^*, c^*, \ldots, c^*)$ is played. Since every player in the same population plays the same action, any stationary equilibrium must be symmetric.

Stationary

Communication among player does not exist. Thus, no player can observe what others do, or the state of the society. He only knows his private history.

As a first step of computing a (sequential) equilibrium, let us focus on a symmetric stationary equilibrium in which each player's action is conditioned on his state instead of entire history.

$$\Sigma_i = \{\emptyset\} \cup \{(j, v_i, v_j) | j \neq i, (v_i, v_j) \in V\}$$

Let W_i^0 and $W_i(v_i)$ be the value functions associated with the states.

Guess an equilibrium outcome where $(r^*, \ldots, r^*, c^*, \ldots, c^*)$ is played. Since every player in the same population plays the same action, any stationary equilibrium must be symmetric.

Let r and c be the index for a representative row and column player.

In an equilibrium, a row player agrees to form a partnership around (v_r, v_c) if

$$W_r(v_r) > W_r^0.$$

We can decompose the value functions as

$$W_r(v_r) = (1-\beta)v_r + \beta\left[\delta W_r(v_r) + (1-\delta)W_r^0\right],$$

and

$$W_r^0 = (1-\beta)v_0 + \beta\left[(1-p^{W^0})W_r^0 + \int_{(v_r', v_c')\geq(W_r^0, W_c^0)} W_r(c, v')d\nu(v')\right].$$

A simple calculation reveals that $W_r(c, v_r, v_c) > W_r^0$ if and only if

$$v_r > W_r^0$$

and

$$W_r^0 = \frac{(1 - \beta\delta)v_0 + \beta p^{W^0} \mathrm{E}[v_r | \mathcal{P}_r]}{1 - \beta\delta + \beta p^{w^0}}.$$

Asymptotic Efficiency

In an equilibrium,

$$(1-\beta\delta)(v_0 - W_r^0) + \beta p^{W^0}\left(\mathrm{E}[v_r|\mathcal{P}_r] - W_r^0\right) = 0$$

must hold. As $\beta\delta \to 1$,

$$\int_{(v_r', v_c') \geq (W_r^0, W_C^0)} v_r' d\nu(v') \to 0$$

must hold, which implies that (W_r^0, W_c^0) converges to the Pareto frontier Hence, any symmetric stationary equilibrium outcome must converge to an efficient allocation.

Pareto frontier

In an equilibrium,

$$\begin{aligned}(1-\beta\delta)(v_r^0 - W_r^0) + \beta p^{W^0}(\mathrm{E}(v_r|\mathcal{P}_r) - W_r^0) &= 0\\(1-\beta\delta)(v_c^0 - W_c^0) + \beta p^{W^0}(\mathrm{E}(v_c|\mathcal{P}_c) - W_c^0) &= 0\end{aligned}$$

must hold where

$$p^{W^0} = \mathrm{P}(v_r \geq W_r^0,\ v_c \geq W_c^0).$$

Our task is to compute (W_r^0, W_c^0) that solves the pair of Bellman equation simultaneously, and show that it converges to the Nash bargaining solution as $\beta\delta \to 1$.

Challenges

Challenges

- As (W_r^0, W_c^0) approaches the Pareto frontier, the "force" tapers off, which used to push the long run average payoff toward the frontier.

Challenges

- As (W_r^0, W_c^0) approaches the Pareto frontier, the "force" tapers off, which used to push the long run average payoff toward the frontier.
- We need to check two vectors:

$$\begin{bmatrix} v_r^0 - W_r^0 \\ v_c^0 - W_c^0 \end{bmatrix}$$

and

$$\begin{bmatrix} \mathrm{E}(v_r|\mathcal{P}_r) - W_r^0 \\ \mathrm{E}(v_c|\mathcal{P}_c) - W_c^0 \end{bmatrix}$$

With little restriction on f_ν, we have little information about the second vector.

Tack

- We prove the conclusion for the case where V is a triangle and f_ν is the uniform distribution over V.
- Suppose that V is a triangle and f_ν is a general distribution satisfying the assumption. Since we know that W^0 is close to the Pareto frontier, we only need to show that the probability distribution around the Pareto frontier can be approximated by the uniform distribution in a certain sense.
- Extend the result to a convex V and a general f_ν satisfying the assumption.

Triangle and uniform distribution

Note that

$$\begin{bmatrix} \mathrm{E}(v_r|\mathcal{P}_r) - W_r^0 \\ \mathrm{E}(v_c|\mathcal{P}_c) - W_c^0 \end{bmatrix}$$

is precisely the vector pointing to the center of gravity of the right triangle $\Delta(W^0)$.

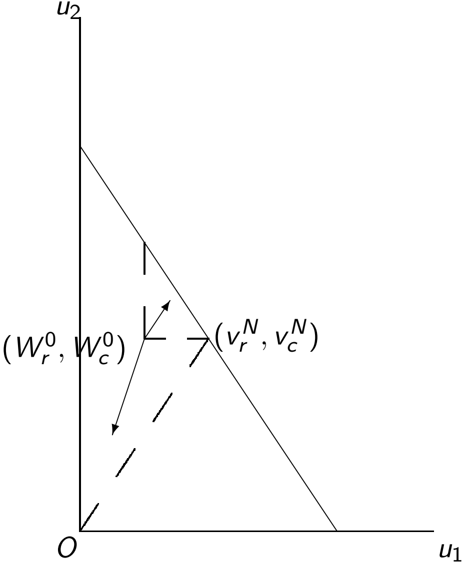
Triangle and uniform distribution

Note that

$$\begin{bmatrix} \mathsf{E}(v_r|\mathcal{P}_r) - W_r^0 \\ \mathsf{E}(v_c|\mathcal{P}_c) - W_c^0 \end{bmatrix}$$

is precisely the vector pointing to the center of gravity of the right triangle $\Delta(W^0)$.

The only (W_r^0, W_c^0) that solves the pair of the Bellman equation must be located along the line connecting (v_r^0, v_c^0) and the middle point of the long edge of V, which is precisely the Nash bargaining solution.


u_2
(W_r^0, W_c^0)
(v_r^N, v_c^N)
O
u_1

General distribution

- The angle between the center of gravity and the expected value depends upon how much f_ν differs from the uniform distribution.
- Since W^0 is close to the Pareto frontier, we only need to consider the case where W^0 is very close, and therefore, the right triangle $\Delta(W^0)$ is small.
- Since f_ν is uniformly bounded away from 0, and is Lipschitz continuous, the deviation from the uniform distribution over $\Delta(W^0)$ is "largely" determined by differences among the values of f_ν at three points of $\Delta(W^0)$.
- As W^0 converges to Pareto frontier, the difference vanishes and the conditional distribution on $\Delta(W^0)$ is approximated by the uniform distribution.

Milder restriction on ν

Instead of f_ν, let us consider a collection of conditional probabilities $f_\nu(\cdot|W^0)$ over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$.

Definition

$f_\nu(\cdot|W^0)$ is locally uniform if $f_\nu(\cdot|W^0)$ converges weakly to a uniform distribution over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$ as W^0 converges to the Pareto frontier of V.

Milder restriction on ν

Instead of f_ν, let us consider a collection of conditional probabilities $f_\nu(\cdot|W^0)$ over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$.

Definition

$f_\nu(\cdot|W^0)$ is locally uniform if $f_\nu(\cdot|W^0)$ converges weakly to a uniform distribution over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$ as W^0 converges to the Pareto frontier of V.

While f_ν does not have to be a uniform distribution, the search over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$ must be sufficiently uniform if W^0 is close to the Pareto frontier.

Milder restriction on ν

Instead of f_ν, let us consider a collection of conditional probabilities $f_\nu(\cdot|W^0)$ over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$.

Definition

$f_\nu(\cdot|W^0)$ is locally uniform if $f_\nu(\cdot|W^0)$ converges weakly to a uniform distribution over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$ as W^0 converges to the Pareto frontier of V.

While f_ν does not have to be a uniform distribution, the search over $\{(v_r, v_c)|v_r \geq W_r^0, v_c \geq W_c^0\}$ must be sufficiently uniform if W^0 is close to the Pareto frontier.

If f_ν fails this property, then f_ν may assign a mass point along the Pareto frontier. Then, the search is very much concentrated at the mass point, and the resulting stationary outcome need not be a Nash bargaining solution.

General case

Extension to a general convex set is straightforward.

General case

Extension to a general convex set is straightforward.

Much, if not all, of the results continues to hold even if the number of agents is finite.

General case

Extension to a general convex set is straightforward.

Much, if not all, of the results continues to hold even if the number of agents is finite. The main role of the continuum of agents is to obtain the threshold rule as the equilibrium decision rule, and the symmetry.

General case

Extension to a general convex set is straightforward.

Much, if not all, of the results continues to hold even if the number of agents is finite. The main role of the continuum of agents is to obtain the threshold rule as the equilibrium decision rule, and the symmetry. Stationary equilibria with the threshold rule exist and converge to the Nash bargaining solution as $\beta\delta \to 1$.

General case

Extension to a general convex set is straightforward.

Much, if not all, of the results continues to hold even if the number of agents is finite. The main role of the continuum of agents is to obtain the threshold rule as the equilibrium decision rule, and the symmetry. Stationary equilibria with the threshold rule exist and converge to the Nash bargaining solution as $\beta\delta \to 1$.

Communication among agents do not exist. We focus on this case to consider the worst possible case to obtain any kind of social coordination. Despite the lack of communication, significant cooperation can be achieved. With some form of communication, the same sort of equilibrium continues to exist.